FISK

Advanced Course in the

Design of Electrical Machines

Electrical Engineering

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ADVANCED COURSE IN THE DESIGN OF ELECTRICAL MACHINES

BY

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THESIS

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

Ira William Fisk

ENTITLED Advanced Course in the Design of Electrical Machines.

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Master of Science in Electrical Engineering

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I ADVANCED COURSE IN THE DESIGN OF ELECTRICAL MACHINES

INTRODUCTION

While very many books have been published on the design of electrical machines, and the market almost flooded with such literature, it has been found impractical to introduce any one of these as a Textbook in the Department of Electrical Engineering at the University of Illinois. The reason for this is, that either the books are highly specialized and deal with only one or two types of machines or otherwise are too general and lack the specific information which is essential to a thorough understanding of the subject. Finally the books are as a rule written by men who, while theoretically qualified to design machines, have had no practical experience.

In this thesis an attempt is made to give a practical course in design, a course which follows closely the lines used by one of the largest manufacturing companies, and which also emphasizes the fundamental principles and theory of these machines.

The author herewith wishes to thank Dr. Ernst J. Berg for his assistance in furnishing the method of attack and machine constants which have been successfully used in the design of commercial machines now in operation.

II

FORM OF MODERN GENERATORS.

The three main parts of the direct current generator may be given as

1. The armature.
2. The field structure.
3. The commutator.

In the designing of these machines an attempt is made to obtain the greatest possible output with a minimum input and to arrive at these results with as small an outlay of material and labor as possible.

In order to produce a given amount of magnetic flux with a minimum expenditure of magneto motive force, the modern generator has developed to the form shown in figure 1 .

In small machines the over all dimensions are governed partly by efficiency in the use of materials and partly by appearance, although practically all field frames conform in appearance to that shown in figure 1 , and also the form of each individual part is affected more or less by the kind of material used. In case of field frames and magnets, either cast iron, cast steel, wrought iron or malleable iron each has its own peculiar magnetic property and thus must be dealt with accordingly.

Cast iron is a material low in permeability but also low in cost. While cast steel and wrought iron are both high permeability materials and also high in cost, although the increased cost is very often justified by the greater ability of these materials to carry flux at a higher density, thus decreasing the weight of a machine and also where the magnetic circuit is to be surrounded by

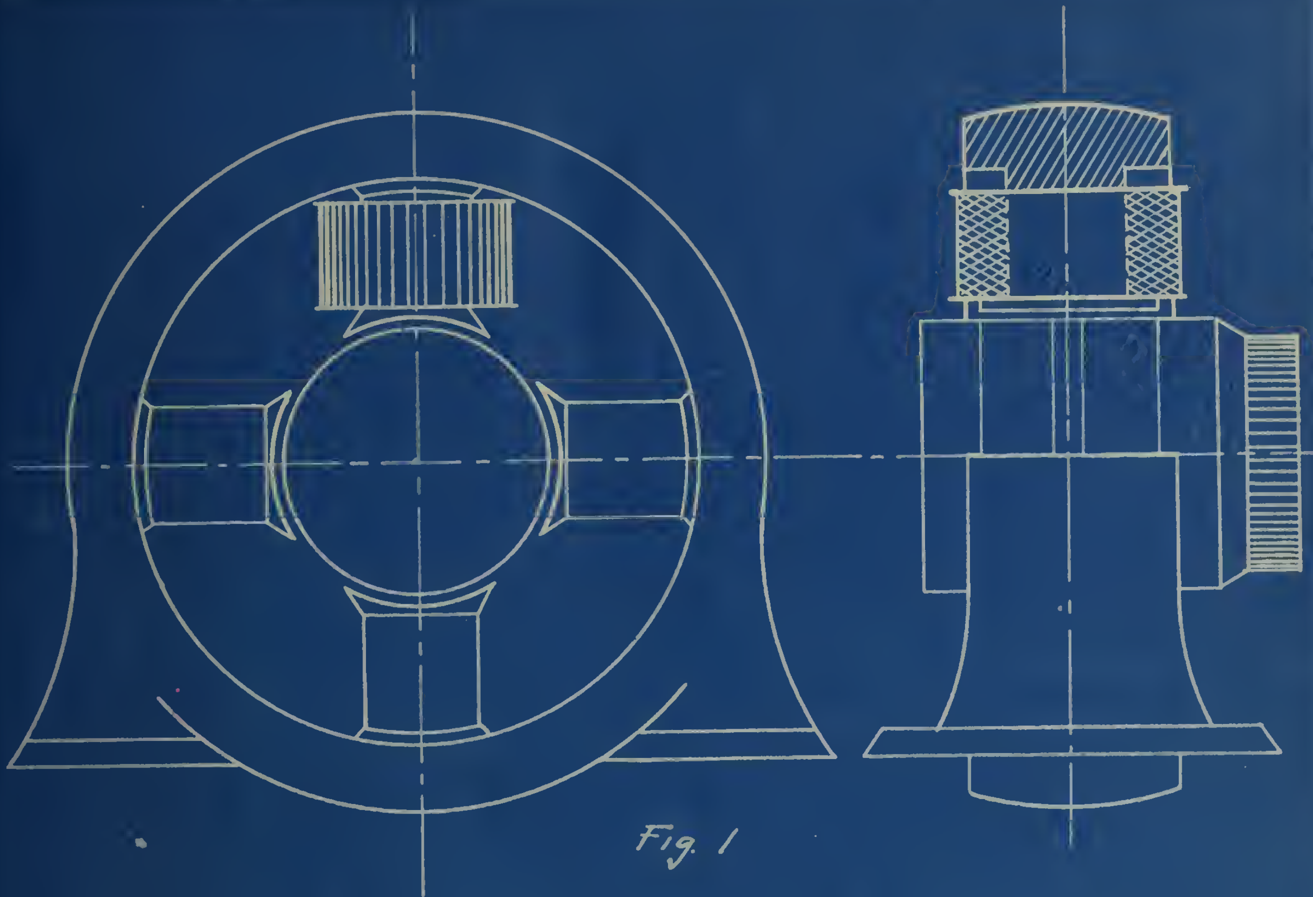


Fig. 1

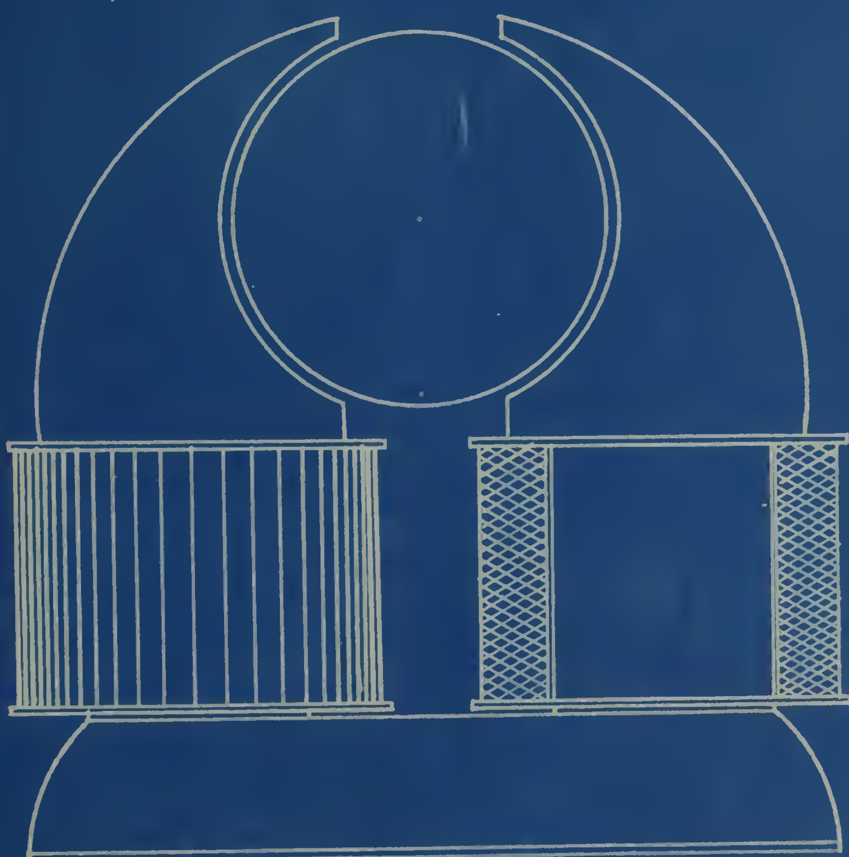


Fig. 2

copper the reduced area will mean a great reduction in the more expensive copper. The curves figure 3 , will give a comparison in permeability of cast steel, wrought iron and gray cast iron.

Wrought iron does not lend itself to special shapes, but may be used as thin plates and is more often used in this form than in any other, and thus is often used to build up field cores and shoes where the eddy current loss must be reduced.

Cast steel is easily cast in any form and is therefore used largely where pole cores and field frames are cast in one piece.

Field cores are fastened to the yoke or frame in several ways, as for instance,

1. Cast with yoke.
    (a) Both core and yoke cast steel.
    (b) Both core and yoke cast iron.
    (c) Core cast steel, yoke cast iron.
2. Laminated core fitted to cast steel yoke.
3. Laminated core fitted to cast iron yoke.

In all cases a shoe is either cast or fitted to the core so as to reduce the density in the air gap.

From the above it is seen that laminated pole cores must be rectangular and it is also evident that a coil wound around a rectangular core will contain more copper than the same coil wound around a circular pole of the same area, but the rectangular laminated pole will greatly reduce the eddy current loss, which is becoming more and more a factor as the manufacturer reduces the number of slots in the armature discs.

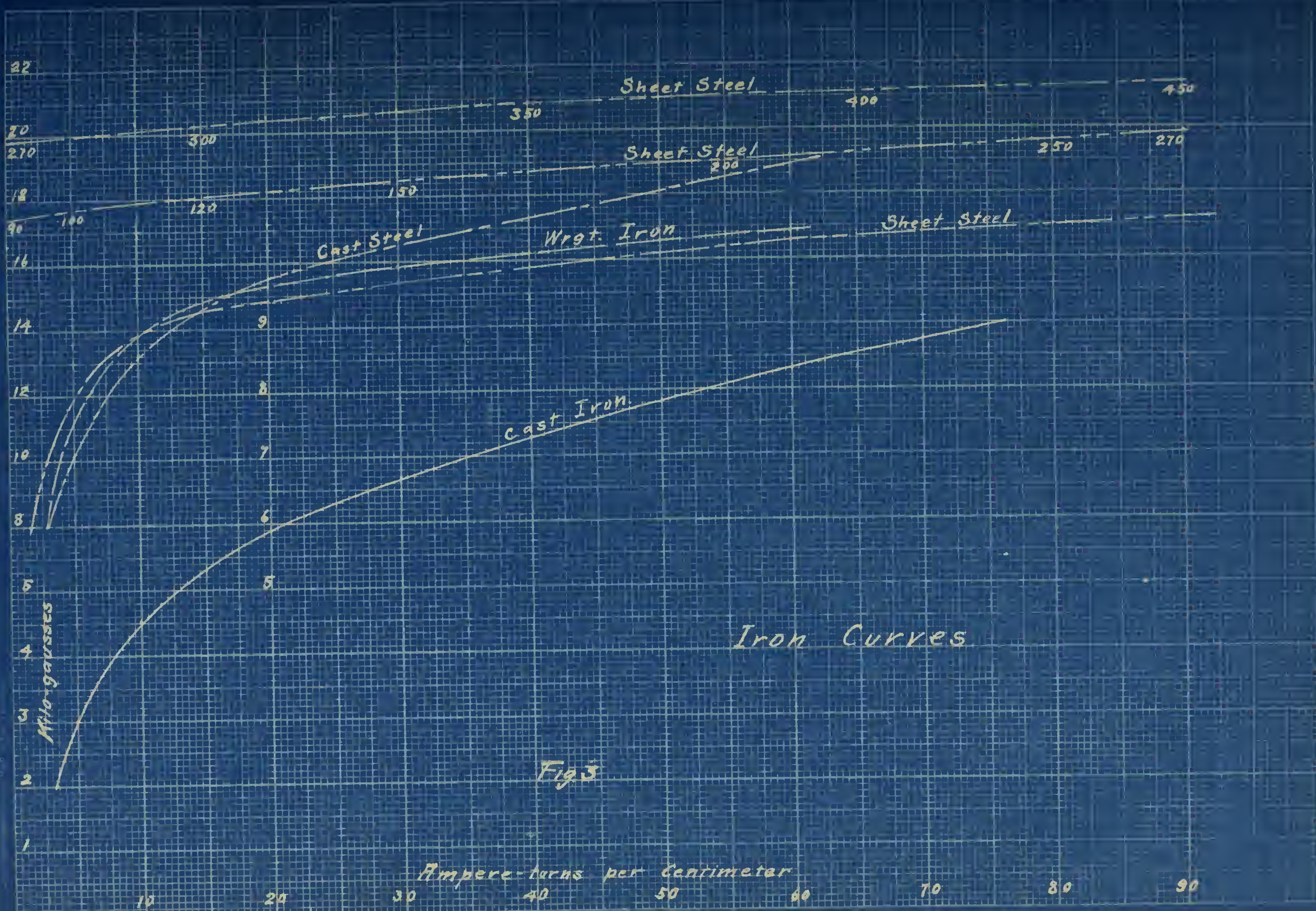

Sheet Steel
Sheet Steel
Sheet Steel
Cast Steel
Wrgt. Iron
Cast Iron
Iron Curves
Kilo-gausses
Ampere-turns per Centimeter

Fig 3

The greater the number of slots the greater will be the amount of insulation used and hence the poorer will be the space factor, and again as the number of slots are reduced the greater will the variation of flux distribution at the surface of the pole shoe and hence the greater the eddy current loss in the shoe and pole core. Hence here we have to work out a balance of conditions to produce the best results with the least cost for materials used.

The ampere turns needed per pole depend almost entirely upon the following,

(a) Length of air gap.

(b) Density in air gap.

as in the modern generator approximately 85% of the total m.m.f. per pole is expended on the gap.

The length of air gap is fixed partly by mechanical considerations and partly by electrical conditions and hence we see that the ampere turns per pole are almost independent of the number of poles.

It is easy to see that the circle has the least perimeter for a given area, so that a comparison may be made of bi-polar and multipolar machines based on the same total flux, watts lost and material used. Therefore the comparison is between an area made up of a large number of small circles and two large circles or between the perimeter of one large circle whose area is "a", and "n" small circles each having an area $\frac{a}{n}$.

Let us take the case of a bi-polar machine and a 2n pole machine of same total flux. Let T = turns per pole in each case.

Area of pole core for bi-polar = a.

Area of pole core for 2n poles = $\frac{a}{n}$.

Diameter of pole core for bi-polar is proportional to $\sqrt{a}$.

Diameter of pole core for n poles is proportional to $\sqrt{\frac{a}{n}}$.

Periphery of pole core for bi-polar is proportional to $\sqrt{a}$.

Periphery of pole core for n poles is proportional to $\sqrt{\frac{a}{n}}$.

Total length of copper for bi-polar will be $\Bumpeq 2T\sqrt{a}$.

Total length of copper for n poles will be $\Bumpeq 2nT\sqrt{\frac{a}{n}}$

Ratio of field copper in a bi-polar to field copper in 2n pole machine is $\frac{2T\sqrt{a}}{2nT\sqrt{\frac{a}{n}}} = \frac{1}{\sqrt{n}}$ (1)

Obviously the tendency would be to build bi-polar machines if we were to consider only field copper. However large machines can not be built as bi-polar machines due to commutation and to magnetic leakage. The above of course relates only to field copper and not to armature copper which would be much greater in the bi-polar than in the multipolar due to the great length of end turn required.

The armature core is built of a very soft laminated steel and has the form of a slotted drum, the armature conductors being placed into these slots.

The commutators of all machines are very similar, all having the same general shape of commutator segment, see page 52 for shape and specifications.

Having decided on the material to be used in the different parts of a machine the dimensions will depend on the following conditions.

Maximum flux densities.

The watts to be dissipated.

The temperature rise.

The flux and current densities to be used are about as follows.

TABLE I.

Density in	Material	Lines per sq. in.
Armature body	Sheet iron or steel stampings	60000 - 95000
Armature teeth	Sheet iron or steel stampings	120000 - 135000
Air gap	Air	20000 - 60000
Pole cores	Cast steel or wrought iron	75000 - 100000
Yoke	Cast steel or wrought iron	70000 - 100000
Yoke	Cast iron	35000 - 50000

CURRENT DENSITY TO BE USED.

	Amperes per sq. in.	sq. mils per ampere.	M ampere
In armature	1500	667	847
	2000	500	637
Commutator risers	2400	417	531
	4000	250	318
Field coils	600	1670	2126
	800	1250	1591
	1000	----	----

In small generators of say 5 kilowatt the air gap density seldom exceeds 30,000 lines per square inch, while in large generators of 500 kilowatt a common figure would be 60,000 lines per square inch.

AIR GAP CALCULATIONS.

For good commutation it has been found that the following ratios should be applied to machines with out interpoles.

(a) $$\frac{\text{Ampere turns per pole to force flux through gap and teeth}}{\text{Ampere turns on armature per pole}} = K.$$

$$K = 0.7 - 1.4$$

(b) $$\frac{\text{Density in the air gap at full load. (lines square inch)}}{\text{Ampere conductors per inch periphery}} = K_1.$$

$$K = \text{approximately } 100 - 150$$

150 being used with machines of small ampere conductors per inch periphery.

(c) $$\frac{\text{Terminal voltage}}{\text{Volts per segment}} = K_2$$

$$K_2 = 20 - 33\ 1/3 \text{ for 100 volt machines.}$$

$$K_2 = 33\ 1/3 - 50 \text{ for 500 volt machines.}$$

Example (b) gives a method for determining the proper density to use in the air gap as a function of the ampere conductors per inch periphery. While case (a) gives a good rule for determining the length of air gap since ampere turns on armature per pole x K = .313 B_g L_g. (2)

Where B_g = density in air gap lines square inch

L_g = length of air gap in inches

DEPTH OF SLOT AS A FUNCTION OF THE DIAMETER OF THE MACHINE ARMATURE.

Let W_s = width of slot.

Let W_t = width of tooth at armature periphery.

t = total number of teeth.

D = diameter of armature.

Then at the armature periphery $W_s + W_t = \frac{\pi D}{t}$ (3)

Let $W_s = M_1 \times W_t$ or $\frac{W_s}{W_t} = M_1$ = a ratio

$$W_t = \frac{\pi D}{(1 + M_1)t} \quad \text{and} \quad W_s = \frac{\pi D M_1}{t\,(1 + M_1)} \tag{4}$$

Let D_s = diameter of the armature measured at the bottom of the slot.

$$\text{Width of tooth at bottom} = \frac{\pi D_s}{t} - \frac{\pi D M_1}{t\,(1 + M_1)} \tag{5}$$

$$= \frac{\pi}{t}\left[\frac{D_s\,(1 + M_1) - D\,M_1}{(1 + M_1)}\right] \tag{6}$$

Let B_t = density at roots of teeth

L' = net iron length of the armature

$$\text{Flux per tooth} = B_t\,L'\,\frac{\pi}{t}\left[\frac{D_s\,(1 + M_1) - D\,M_1}{1 + M_1}\right] \tag{7}$$

Let r_p = ratio pole arc to pole pitch

$$\text{Pole face arc} = \frac{L\,\pi\,D\,r_p}{P} \tag{8}$$

Let B_p = pole face density

$$\text{Flux from pole face} = \frac{B_p\,L\,\pi\,D\,r_p}{P} \tag{9}$$

The number of teeth carrying the flux is $t.\ r_p\ .\ \frac{M_2}{P}$

Where M_2 is greater than unity to allow for fringing.

Flux carried by teeth per pole

$$B_t\,L'\,\frac{\pi}{P}\left[\frac{D_s\,(1 + M_1) - M_1\,D}{1 + M_1}\right] r_p\,M_2 \tag{10}$$

Therefore

$$\frac{\pi\, B_p\, L\, D\, r_p}{P} = B_t \quad L' \frac{\pi}{P} \left[\frac{D_s\,(1 + M_1) - M_1\, D}{1 + M_1} \right] r_p\, M_2$$

$$\frac{B_p\, L\, D}{B_t\, L'\, M_2} = \frac{D_s\,(1 + M_1) - M_1\, D}{1 + M_1} \tag{11}$$

Putting $\dfrac{B_p\,L}{B_t\,L'\,M_2} = K$ and transposing

$$K\,D\,(1 + M_1) = D_s\,(1 + M_1) - M_1\,D \tag{12}$$

$$K\,D\,(1 + M_1) + M_1\,D = D_s\,(1 + M_1) \tag{13}$$

$$D\left[K(1 + M_1) + M_1\right] = D_s\,(1 + M_1) \tag{14}$$

$$\frac{D}{D_s} = \frac{1 + M_1}{K\,(1 + M_1) + M_1} \tag{15}$$

$$D = \frac{(1 + M_1)\,D_s}{K\,(1 + M_1) + M_1} \tag{16}$$

$$D_s = \frac{D\left[K(1 + M_1) + M_1\right]}{(1 + M_1)} \tag{17}$$

$$\frac{D - D_s}{2} = \frac{(1 + M_1)\,D_s}{K\,(1 + M_1) + M_1} - \frac{D\left[K(1 + M_1) + M_1\right]}{(1 + M_1)}$$

and $$\frac{D - D_s}{2} = \frac{1 - K\,(1 + M_1)}{2\,(1 + M_1)} \times D \tag{18}$$

K must be less than $\dfrac{1}{1 + M_1}$

As an example $\dfrac{B_p}{B_t} = \dfrac{1}{2.7} = \dfrac{50000}{135000}$

$\dfrac{L'}{L} = .8 \qquad M_2 = 1.1 \qquad M_1 = 1$

$$\text{Then } K = \frac{B_p}{B_t\ M_2}\ \frac{L}{L'} = \frac{50000 \times 1.25}{135000 \times 1.1} = \frac{L'}{L} = 1.25 \qquad (19)$$

$$K = .42$$

Depth of slot = .04D

This value of depth of slot is usually exceeded in small machines or machines with relatively small diameter and the ratio M_1 is seldom taken as unity but some factor greater than unity. Also this formula is based on the assumption that all the flux entering the armature passes through the teeth an assumption that is far from true especially at high densities as will be shown below.

CORRECTION FOR FLUX DENSITY IN ARMATURE TEETH.

When low densities are used in armature teeth it is evident that all or very nearly all the flux entering the armature from a pole piece will take this path rather than pass through the slot which has a much lower magnetic conductivity.

If we let a = width of a tooth

b = width of a slot

K = effective length of the armature say = .75 of the total length

L = depth of slot

Then area of one tooth = Ka

$$\text{Then area of one slot} = \frac{K\ b}{.75} + \frac{.25\ Ka}{.75} \qquad (20)$$

Assume φ = total flux to be passed through tooth and slot

φ_1 = flux through the slot.

Then $\varphi - \varphi_1$ = flux through one tooth, and since the conductance of any magnetic circuit is $\frac{\mu \times \text{area}}{\text{length}}$ we have the

$$\text{Conductance of one tooth} = \frac{K\,a\,\mu}{L}$$

$$\text{Conductance of one slot} = \frac{1.34\,b\,K + .33\,K\,a}{L}$$ since for air $= 1$ and therefore the total flux will divide and pass through the two paths according to their conductances, or

$$\frac{\varphi_1}{\varphi - \varphi_1} = \frac{\dfrac{1.34\,K\,b + .33\,K\,a}{L}}{\dfrac{K\,a\,\mu}{L}} = \frac{1.34\,b + .33\,a}{a\,\mu} \qquad (21)$$

and from the above equation

$$\varphi = \frac{\varphi_1\,(a\mu + 1.34\,b + .33\,a)}{1.34\,b + .33\,a}$$

If we assume all the flux passes through the teeth then we can find the ratio of apparent flux density to true flux density.

$$\text{Or} \quad \frac{\varphi}{\varphi - \varphi_1} = \frac{\dfrac{\varphi_1\,(a\mu + 1.34\,b + .33\,a)}{1.34\,b + .33\,a}}{\dfrac{a\,\mu\,\varphi_1}{1.34\,b + .33\,a}}$$

$$\frac{\varphi}{\varphi - \varphi_1} = \frac{a\mu + 1.34\,b + .33\,a}{a\,\mu} \qquad (22)$$

Now let B = the density in teeth assuming all the flux passes through the teeth, and let B_1 = true density when $\varphi - \varphi_1$ = flux through teeth. Then B is proportional to φ and B_1 is pro-

portional to $\varphi - \varphi_1$ and $\dfrac{\varphi}{\varphi - \varphi_1} = \dfrac{B}{B_1} = \dfrac{a\mu + 1.34\,b + .33\,a}{a\mu}$ (23)

By inspection of equation 23 it is seen that for low values of B, μ will be very large in proportion to 1.34 b + .33 a and hence B_1 will be very nearly equal to B, but as the density becomes very high as in railway motors B_1 will be considerable less than B and therefore the correction is small for moderate flux densities and large for high flux densities.

NUMBER OF TEETH.

For economy the fewer the teeth the better, while for a good flux distribution at the pole face a large number of teeth per pole are required. Again, when few teeth are used a large number of coils per slot will be required, and trouble may be introduced due to position of the slot with respect to the pole when the first or last coil in this slot is commutated. This is especially true in machines using a large ratio $\frac{\text{pole arc}}{\text{pole pitch}}$. A rule that is often applied is to make the number of slots equal to four times the diameter of the armature in inches, which means the slot pitch will be $\frac{\pi}{4}$ i.e. width of slot and tooth at armature face = $\frac{\pi}{4}$ = approximately .8". It is evident of course that for small motors such as one-fourth to one horse power that this rule will not hold since .8" slot pitch in the small sizes of motors or generators would be too large to give a reasonable flux distribution under the pole face.

CALCULATION OF FIELD AMPERE TURNS.

Let A_y = area of yoke perpendicular to lines of force.

Let A_c = area of core perpendicular to lines of force.

Let A_g = area of gap perpendicular to lines of force.

Let A_t = area of teeth perpendicular to lines of force.

Let A_a = area of armature perpendicular to lines of force.

Let B_y = density in yoke lines per square inch.

Let B_c = density in core lines per square inch etc.

Let L_y = length of magnetic path in yoke per pole.

Let L_c = length of magnetic path in core etc.

The length for a given machine may easily be calculated and the areas of the various sections depend on the material used and on the total flux to be carried, the proper density has been given on page 5 .

The area of the air gap may be calculated by assuming its area to be

Area of pole face + arc of teeth under a pole

divided by 2 or

$$\frac{A_p + A_t}{2}$$ where A_t = area(under one pole)of teeth at armature face. Some designers add 10% for fringing as the above expression assumes that the flux passes in straight lines from the pole face to armature teeth. A more correct relation is

Pole arc + air gap length x constant.

This constant depends chiefly upon the ratio

$$\frac{\text{distance between pole shoes}}{\text{length of air gap}} . \qquad (24)$$

TABLE II

distance between shoes / length of gap	4	5	6	7	8	9	10	14
Constant	1.32	1.59	1.79	1.98	2.15	2.3	2.43	2.84
distance between shoes / length of gap	18	22	24	26	28	30	40	50
Constant	3.15	3.4	3.51	3.61	3.7	3.78	4.14	4.4

And area of air gap using these constants will be

Effective pole arc = Pole arc + constant x L_g

Effective area of pole shoe =

Pole arc + constant x L_g x length of pole shoe = A_s

$$\text{Air gap area} = \frac{A_s + A_t}{2} \quad (25)$$

LEAKAGE FACTOR.

Since the reluctance of the air gap is high it is evident that a portion of the flux generated in the pole core will pass between poles and not enter the armature body. Thus it is evident that for a given armature flux a greater flux must be generated by the field ampere turns and must be carried by the pole core and yoke as illustrated in figure 4 .

$$\text{The ratio} \quad \frac{\text{Flux in field core}}{\text{Flux entering armature body}} = \lambda$$

where λ is known as the leakage factor.

TABLE III

Approximate leakage factors.

Killowatts	λ	Killowatts	λ
5	1.25 - 1.4	50	1.15 - 1.25
10	1.25 - 1.35	100	1.1 - 1.2
25	1.2 - 1.3	200	1.08 - 1.15

For any given armature flux we may now find the flux carried by any section of the generator and hence may settle the density. From the magnetization curves we can find the ampere turns necessary per unit length to produce this flux density and by multiplying by the length of path in each case we may find the

correct number of ampere turns per field pole.

MAGNETIZATION CURVE.

The magnetization curve for any machine may be found by the same process since the voltage will increase directly with the armature flux and for any armature flux we can calculate the necessary ampere turns per pole

CALCULATION OF SIZE OF SHUNT FIELD WIRE.

Let IT = required number of ampere turns per pole.

Let E = terminal voltage of the generator.

Let R = field resistance all coils in series.

Then $I = \frac{\frac{3}{4}E}{R} = \frac{3}{4}\frac{E}{R}$ assuming 75% of the terminal voltage to be consumed across the field windings.

Now let L = mean length of one turn in feet and N = number of poles.

$$I = \frac{3}{4}\frac{E}{R} \qquad R = L \times T \times r_1 \times N$$

$$I = \frac{.75\ E}{L \times T \times r_1 \times N}$$ where r_1 = resistance of conductor per foot.

Or $$r_1 = \frac{.75\ E}{L \times IT \times N} \tag{26}$$

If r_1 = resistance of conductor per 1000 feet then

$$r_1 = \frac{750\ E}{L \times IT \times N}$$ which determines the size of field wire to be used for any given E.M.F. and number of ampere turns, the only other factor entering is the watts lost per square inch of ra-

diating surface of the coil as will be shown later.

For small machines not above 250 volts single cotton covered wire provides sufficient insulation and .01 inch must be added to the diameter of the wire, this will allow not only for the covering but also for any irregularities in the wire. For larger machines and higher voltages we usually use double cotton covered wire and .018 inch must be added to its diameter.

The turns per layer will therefore be

$$\frac{\text{length of winding space}}{\text{diameter of covered wire}}.$$

Number of layer = $\frac{\text{depth of coil}}{\text{diameter of wire}}$ and cross section area = (diameter of wire)^2 x number of turns per coil.

Space factor, round conductors, no bedding.

Let d_1 = diameter of bare wire.

Let d = diameter of wire insulated.

Area of copper = $\frac{\pi d_1^2}{4}$.

Space occupied by each conductor = d^2

$$\text{Space factor} = \omega = \frac{\pi}{4}\frac{d_1^2}{d^2} = .7854 \frac{d_1^2}{d^2} \qquad (27)$$

Consider extreme case of bedding.

Area of bare conductor = $\frac{\pi d_1^2}{4}$.

Space occupied by each conductor will be a hexagonal as shown in figure 6 and the area of this hexagonal is found to be

$$12 \times \frac{d}{4} \times \frac{d}{2} \tan 30 = .86\, d^2$$

$$\text{Space factor } \omega = \frac{.7854\, d_1^2}{.86\, d^2} = .906 \frac{d_1^2}{d^2} \qquad (28)$$

For rectangular conductors the space factor depends only on the thickness of insulation as the winding space is completely filled. The radiation of heat is also much better when rectan-

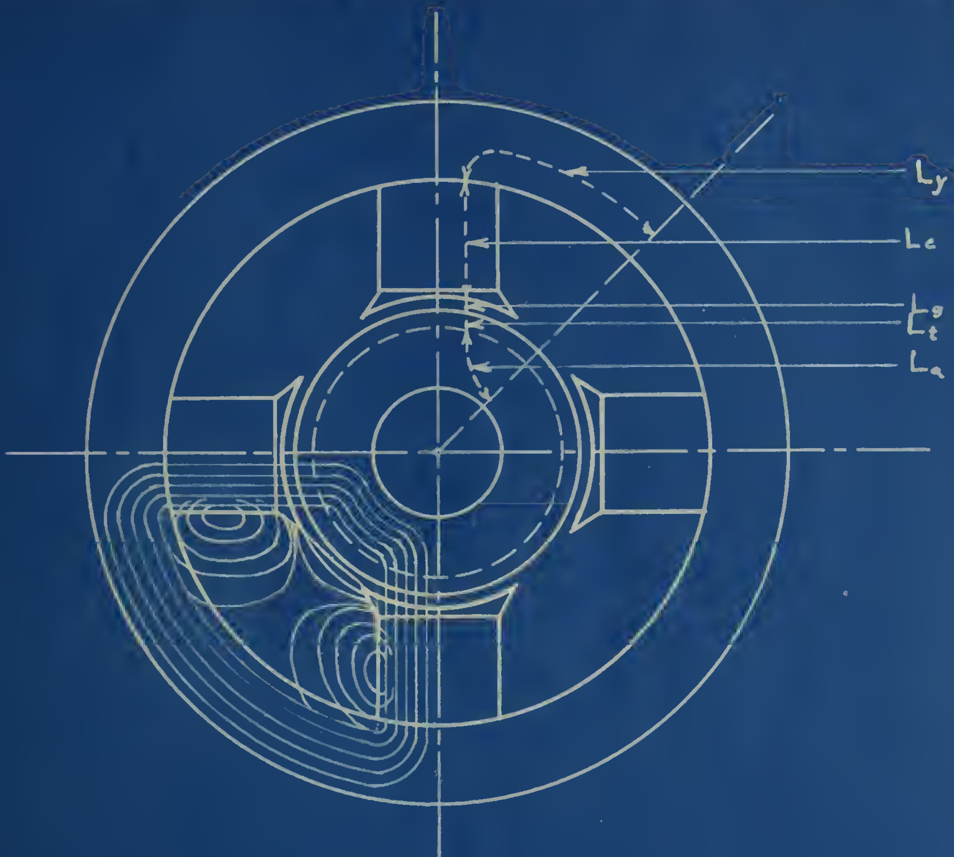


Fig. 4

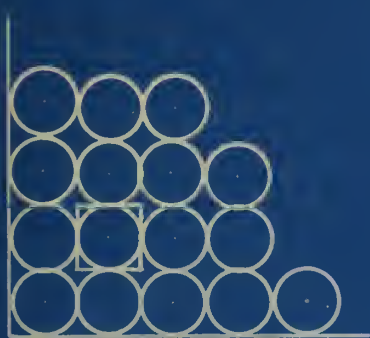


Fig. 5

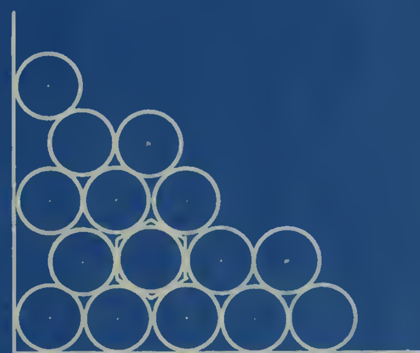


Fig. 6

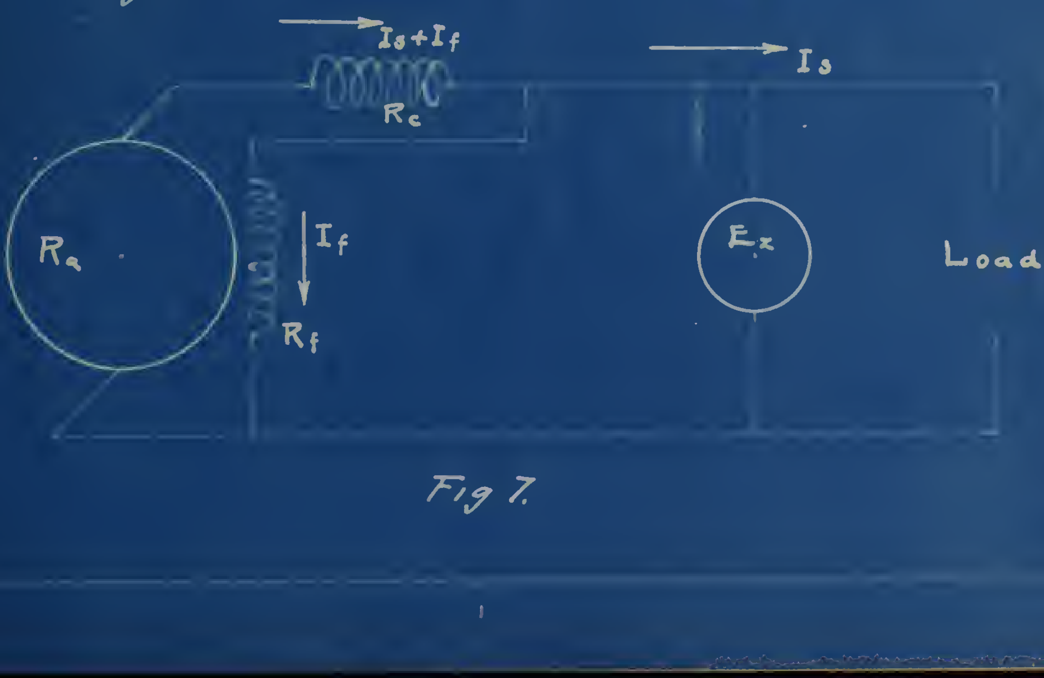


Fig 7.

gular wire is used, since there will be no air spaces between turns, air being a very poor conductor of heat. Therefore square or rectangular conductors should be used whenever possible.

SERIES FIELD CALCULATION.

When a generator is to be built with a compound winding it is necessary to predetermine approximately the number and size of these turns. This may be done provided the full load voltage be known.

Let E_1 = no load voltage and E_2 = the full load voltage at the terminals of the generator.

Let I_f = shunt field current.

Let I_s = load current.

Let R_a = resistance of the armature.

Let R_c = resistance of the series field.

Let R_f = resistance of the shunt field.

When full load line current is flowing we have E_2 as terminal e.m.f. Therefore we must generate in the armature of the machine not only a voltage = E_2, but also

$$(I_s + I_f)\ (R_a + R_c) + \text{brush drop.} \qquad (29)$$

The brush loss will be shown to be 1.8 x $(I_s + I_f)$ watts = $(I_s + I_f)^2 \times R_b$.

Therefore total voltage generated will be

$$E_2 + (I_s + I_f)\ (R_a + R_c) + 1.8 \qquad (30)$$

By substituting this value of e.m.f. in the formula for electromotive generated we find the value of flux which must be produced to give the new voltage.

By the same method as used for calculating the shunt

ampere turns we can find the mew value of ampers turns = A T_f, for full load. Now since the E.M.F. at full load is greater than at no load and, since the shunt field is connected directly across the terminals of the machine, the shunt ampere turns at full load will be increased in the same proportion. See Fig. 7.

Hence the series ampere turns will be

$$A\ T_f - A\ T_s = A\ T_c \tag{31}$$

Where $A.T_f$ = full load ampere turns required, $A.T_s$ = ampere turns (shunt field) full load and A T_c = ampere turns to be provided by the series field neglecting armature reaction. Therefore to calculate the total series ampere turns ; add to A T_c the number of armature conductors between adjacent pole horns multiplied by the current in each conductor.

Let Z. = conductors between adjacent pole horns and I. = current in each conductor. Then total series ampere turns will be (A T_c + I.Z.)

In machines of large output it is standard practice to carry only a portion of the total current through the series winding the remainder being passed through a shunt around the series winding. This shunt is made adjustable and the machine can be made to flat or over compound by simply adjusting the shunt. This also avoids the use of very large series field conductors.

THE GENERATION OF ELECTRO-MOTIVE FORCE.

When a conductor is moved across a magnetic field an electro-motive force is generated therein. The value of this E.M.F. is proportional to the rate of cutting lines of force or

$$e = \frac{d\varphi}{dt} = \text{flux cut per unit of time.}$$ See Fig. 8.

Suppose the coil of wire represented by a.b.c.d. is rotated in the magnetic field. Then for every revolution the flux Φ will be cut by the conductors four times and if one revolution is made per second the average electro-motive force induced therein will be

E average = 4 Φ Now if the coil contains n turns

E average = 4 Φ n and if the coil makes S revolutions per second

$$E \text{ average} = 4 \Phi n S \text{ in abvolts.} \tag{32}$$

If the magnetic field and the speed of rotation is uniform the E. M. F. at the terminals of the coil at any instant may be represented by a sine curve as shown in figure 9 .

Where T. represents the time of one revolution or time of one complete cycle therefore one revolution per second will represent one cycle per second, and since frequency, f, is defined as the cycles per second we may substitute in equation 32, f for S and obtain

$$E \text{ average} = 4 \Phi n f \text{ in abvolts or}$$

$$E \text{ average} = 4 \Phi n f 10^{-8} \text{ in volts.}$$

Where Φ = total flux per pole

n = furns in series.

f = cycles per second.

This formula will hold for any number of poles provided Φ is designated as the flux per pole, and since

$$f = \frac{P \times S}{2} \tag{33}$$

We may write

$$E \text{ average} = \frac{4 \Phi n P S}{2 \times 10^8} \tag{34}$$

where n = turns in series as above.

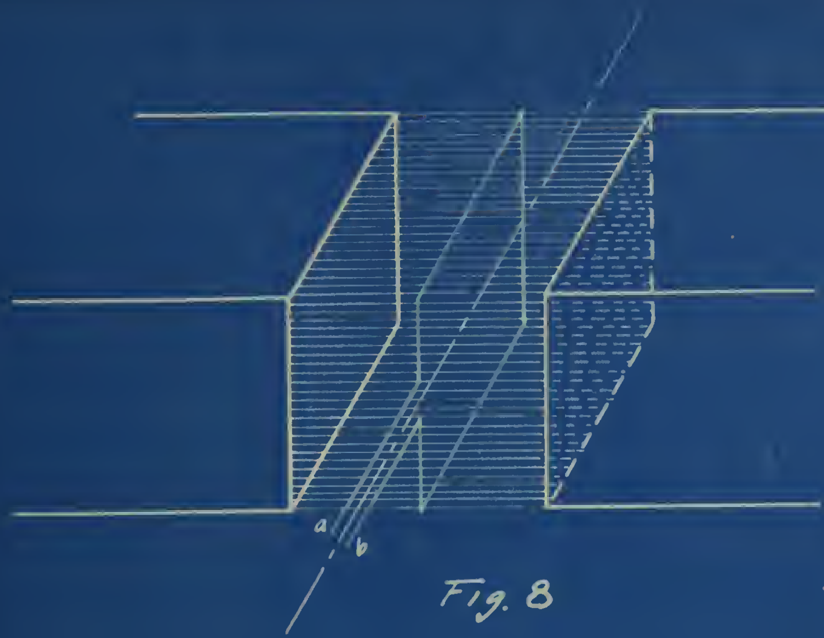


Fig. 8

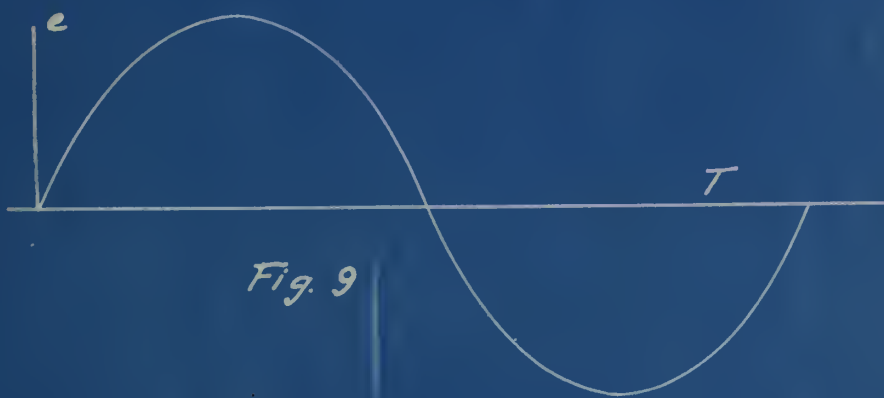


Fig. 9

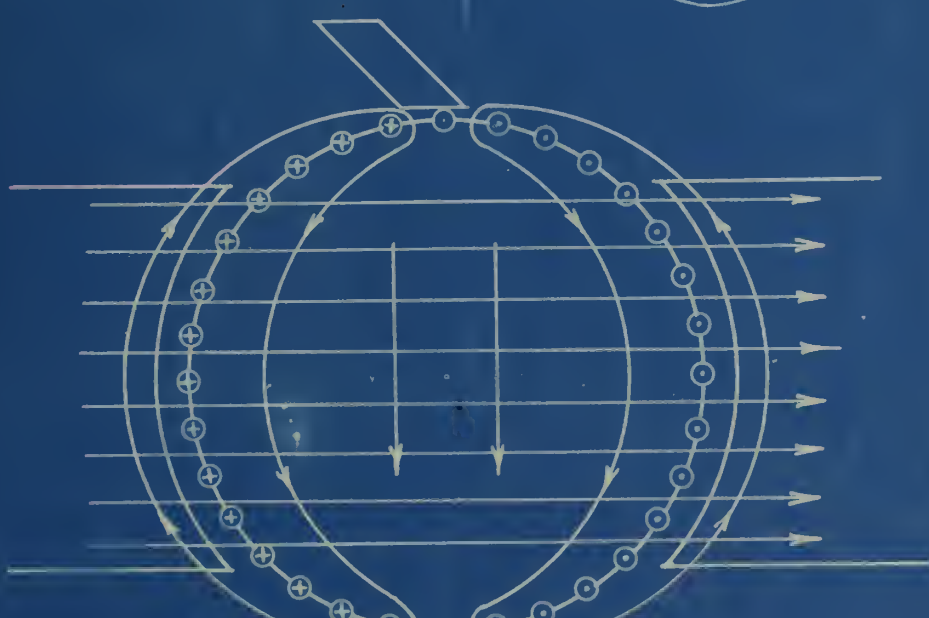


Fig. 10

In all direct current generators there are at least two groups of windings in multiple and there may be as many as there are poles. Therefore if we let Z = the total conductors on the armature and since $\frac{Z}{2} = n$, the equation becomes

$$E \text{ average} = \frac{\Phi Z S P}{P' \times 10^8} \qquad (35)$$

Where P' = the number of paths through the armature, or $\frac{1}{P'} \times Z$ = number of conductors in series on the armature.

Thus we have the general expression

$$E \text{ average} = \frac{\Phi Z S P}{P' \times 10^8}$$ which may be applied to any type of armature winding.

When the conductors of the armature are carrying current a magneto motive force is set up which produces a flux at right angles to the field flux as shown in figure 10 , and therefore a resultant flux is produced in the armature as shown in figure 11 .

It would seem that since the resultant field is greater than the original field the electro-motive force generated would be greater, however this is not the case as only the flux at right angles to the brushes can produce an electro-motive force at the brushes.

So long as the brushes remain in the neutral position i.e. (in such a position that the armature field is at right angles to the main field) the strength of the main field is practically unchanged, but the instant the brushes are shifted in either direction from the neutral axis there is set up a component of the armature turns in such a direction as to weaken or strengthen the main field.

For the proper operation of a generator the axis of the coils undergoing commutation should be at right angles to the resultant field.

With the brushes in the position shown in figure 11 and assuming the reluctance of the main field = the reluctance of the armature field, we have

$$I\,T_r = \sqrt{\overline{I\,T_f}^2 + \overline{I\,T'_a}^2} \qquad (36)$$

If we move the brushes forward as shown in figure then the direction of the armature field is also changed. The resultant field will also change from that shown in figure 11 and it would seem that the brushes would never reach a position such that the axis of commutation is at right angles to the resultant field, but in actual practice this is often accomplished.

Suppose we move the brushes forward to some position in advance of that shown in figure 11 . The angle β would then be greater than a right angle and the brushes would not be in the neutral position as $I\,T_a$ and $I\,T_f$ are considered as constant in value.

From figure 12 it is seen that $I\,T_r$ is less than $I\,T_f$ besides being shifted by the angle ω and hence if the brushes are in the true neutral position

$$\overline{I\,T_f}^2 = \overline{I\,T_a}^2 + \overline{I\,T_r}^2 \qquad (37)$$

or

$$\overline{I\,T_r}^2 = \overline{I\,T_f}^2 - \overline{I\,T_a}^2 \qquad (38)$$

and ω = the angle through which the brushes must be shifted to be in the neutral position,

$$= \sin^{-1} \frac{I\,T_a}{I\,T_f}$$ thus if we know $I\,T_a$ and $I\,T_f$, ω may be determined.

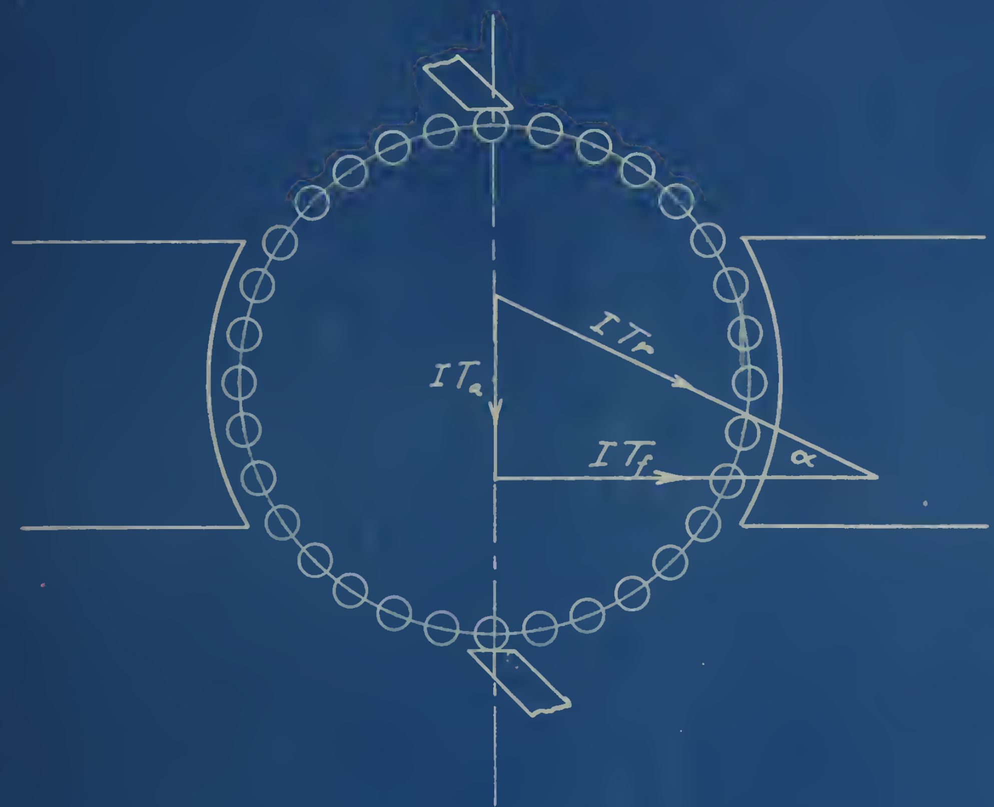


Fig. 11

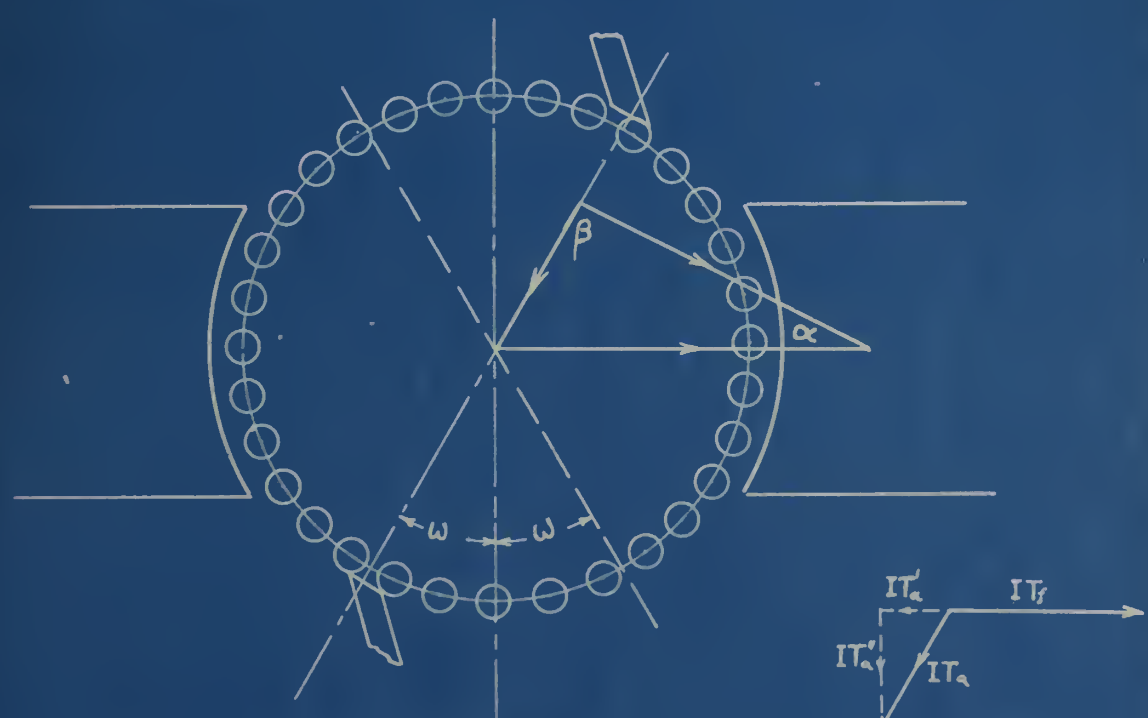


Fig. 12

Although this method shows clearly what happens in the armature of the machine the calculations should not be relied upon unless the effectiveness of the field and armature ampère turns are known, since each will have a different reluctance.

It is usually better to consider the armature ampere turns as split up into two magneto motive forces, or two solenoids half the conductors being on each side but split up into two parts as indicated in figure 12 . The small solenoid composed of the conductors in the double angle ω at the top and bottom of the armature form one solenoid and the remaining conductors on each side of the armature form the second solenoid. It is evident from the diagram that the small solenoid will produce a flux opposed to the field flux and the large solenoid will produce a flux at right angles to the field flux. Therefore the former ampere turns $I'T_a$ are called shifting or distorting ampere turns.

Let T_a = total armature turns. Let ω = the angle through which the brushes are shifted.

The demagnetizing ampere turns will be

$$\frac{2\ T_a \times 2\ \omega}{360} \qquad (39)$$

and for an n pole machine the current per turn = $\frac{\text{total current}}{n}$ = $\frac{I_e}{n}$, then the demagnetizing ampere turns per pole will = $\frac{4T_a\ \omega\ I_e}{360 \times n}$ and therefore the shifting ampere turns will be the difference between total ampere turns per pole and $\frac{T_a\ \omega\ I_e}{90 \times n}$. (40)

In calculating the field ampere turns necessary to overcome the demagnetizing action of the armature the brushes are assumed to be shifted until 2ω = the distance between the pole shoes as has been stated before and the total ampere turns on the field

will then be

$$I\ T\ total = I\ T_f + \frac{T_a\ \omega\ I_e}{90\ n}$$ where I_e = total or

line current.

INSULATION.

In choosing the insulation to be used for electric machinery great care must be taken to obtain not only a great dialetric strength but also mechanical strength and it is an unfortunate fact that the insulations that possess good dialetric qualities usually have poor mechanical properties.

The chief materials which may be used due to their mechanical rather than electrical strength are given below with values of their disruptive strengths.

	Eff.volts.	Disruptive
Brown paper .04" thick	7000	
Red rope paper .04" think	6800	
Express paper .04" thick	7000	
Manila paper .04" thick	6000	
Press spahn paper .04" thick	9000	Dry
Horn fiber paper .04" thick	6000	
Dry wood (maple) .04" thick	6000	

None of these materials have a very high disruptive strength due to the fact that they will all absorb moisture.

Besides these many other insulations are used such as cotton and flax fabrics which are made up and impregnated with insulating compounds and sold under trade names as (1) varnished linen, (2) varnished long-cloth, (3) varnished canvass, (4) Empire cloth.

These materials have a disruptive strength of 7000-10000

eff. volts per ten mills thickness. They have great mechanical strength and are very flexible.

Of materials possessing a high dialetric strength and poor mechanical quality the best known is "mica". The disruptive strength of this material varies greatly and has been found to be from 20,000 to 200,000 eff. volts per .04" thick. Furthermore it is practically fire proof and suffers only when bent sharply or when so placed as to loose its thickness due to flakes being torn off.

To over come this it is made up in various special forms by being cemented on to a backing, a mica-paper, mica-long cloth, or mica canvas. It is also made up with shellac or other cement into micanite and as such while hot may be molded into any form desired. The following table will give comparative disruptive strengths:

Material	Disruptive strength eff. volts.
Micanite .04" thick	30,000
Mica canvas .04" thick	2,500
Mica cloth .04" thick	2,500
Mica paper .04" thick	15,000 -- 20,000

PORCELAIN- See transformer design.

Many rubber compounds have been and are still used, such as vulcanite, and ebonite. These especially the latter are excellent insulators, but they suffer the disadvantage that if made flexible they soften at a very low temperature and if made hard they are usually brittle.

INSULATION OF ROUND WIRE.

Insulated wire is made by spinning over the wire a coating of cotton or silk. Cotton covered wires are made in three grades

(1) Single cotton covered S.C.C.

(2) Double cotton covered D.C.C.

(3) Tripple cotton covered T.C.C.

Silk covered wires are usually double covered only.

The thickness of these insulations vary somewhat with the diameter of the wire, but as can be seen from the wire table the variation is small and may be found for any size by subtracting the diameter bare from the diameter cotton covered as given in table.

TEMPERATURE RISE IN FIELD COILS.

It is evident that the heat generated in a field coil due to its I^2R loss must be radiated from its surface and we have therefore the following conditions.

(1) That the surface for cooling to be taken into account should be the whole surface i.e. external cylinder of coil, internal cylinder of coil and both ends.

(2) That these surfaces are not all equally valuable as radiators.

(3) That the surface exposed to free air and those next to metal if exposed to circulation of air are about equal as radiators and are the best for heat dissipation.

(4) That any surface so near the iron core, pole shoe, or yoke

that no air is allowed to circulate is only about one-half as valuable as surfaces exposed to circulating air.

(5) That the temperature rise depends upon the thickness of the coil and also on the amount of insulating material wrapped around the coil.

The formula for maximum temperature rise may be written as $T = K \frac{\text{watts}}{\text{area}}$ or $T = K \frac{W}{A}$ (41) T = deg. C.

Where W = watts lost in coil, A = sum of all surfaces included under (3) and one-half the surfaces under (4). A good safe value of K = 100.

TEMPERATURE RISE ARMATURE.

Causes,

(a) $I_a^2 R_a$ loss in armature windings.

(b) Eddy current losses in armature conductors. This loss is usually small and almost impossible to calculate.

(c) Iron losses in armature core. Especially in the teeth of toothed armatures these losses may be calculated or taken from curves.

(d) Friction losses in bearings.

As in the case of field coils the heat radiated from the armature depends upon the heat radiating surface and also on the peripheral speed at which the armature is running.

One formula which has been much used is

$$T = \frac{a}{1 + b\,v} \times \frac{P}{A} \qquad (42)$$

a and b are constants, see Fig. 13.

Fig. 13

Fig 13 a

v = peripheral speed in feet per min.

P = watts lost in armature = $I_a^2 R_a$ + core loss.

A = area radiating surface.

The whole worth of this formula depends upon the method of calculating A.

Two ways of calculating A will be given,

(1). The most common is, A = π (diameter of armature x length over end connections)

(2). That which takes into consideration the ventilation ducts. Where A is made up of the $\sum$ of all the shaded areas.

For method (1) b = approximately .0005, a = 45 - 20.

The former value is taken for small machines as those with armature diameters up to eighteen inches where the internal diameter is small and nearly filled with shaft and spider.

For method (2) a = 45 - 90 and b = .0005 for well ventilated machines.

There are many other methods in use such as watts per square inch radiating surface, using as this surface π x diameter x length of armature over all. Where watts per square inch must not exceed a certain value approximately 1 watt.

ARMATURE WINDINGS.

There are two general types of armature windings, i.e. the open coil and closed coil. The first type is used only for arc lighting generators where a high voltage and constant current is desired and since this machine is being rapidly replaced by the mercury arc rectifier no further mention will be made concerning the open coil armature.

For direct current generators that are at the present time in use,many forms of closed coil windings are employed, but in this paper only the multiple circuit and two circuit type will be described.

The closed coil armature apart from the commutator, forms an endless winding, i.e. starting from any point in the winding one would trace through the complete winding before returning to the starting point. This is said to be a singly re-entrant winding whereas a winding formed of two or more independent endless windings each closing upon itself, is said to be a multiply re-entrant winding.

Thus in a doubly re-entrant winding there are two separate windings each closing upon itself.

Again a closed circuit winding can have no less than two circuits in parallel, each of these will have the same number of turns, and the current entering one brush will divide equally through the two halves to unite again at the other brush.

There may however be more than two paths for a singly re-entrant winding and of course there will be more than two paths for a multiply re-entrant winding.

Close coil armature windings may again be divided into drum and ring windings.

RING ARMATURE.

In order to make later explanations more clear reference will here be made to the ring armature which at the present time is nearly obsolete.

The simple Gramme ring wound armature consists of a series of spiral winding, the beginning of the first being connected to the end of the last, thus forming a closed winding. At equal distances around the periphery leads are brought out and connected to the commutator and the brushes for a bi-polar are placed at the neutral as shown in figure 14 .

If a = cross section of the armature wire in square inches,

L = total length of wire in inches.

Then the resistance of the wire all in series will be $\frac{\rho L}{a}$ where ρ = specific resistance of copper at some definite temperature. But the resistance between the positive and negative brushes will be $\frac{\rho L}{4 a}$ since the length L is divided into two paths and these paths are connected in multiple as shown by figure 14 .

From this equation we may derive the general equation for the resistance of a direct current armature, i.e.

Resistance of armature = $\frac{\rho L}{n^2 a}$ where

L = length of all wires connected in series.

ρ = specific resistance at a given temperature.

a = cross section area of series conductor.

n = number of paths through the armature.

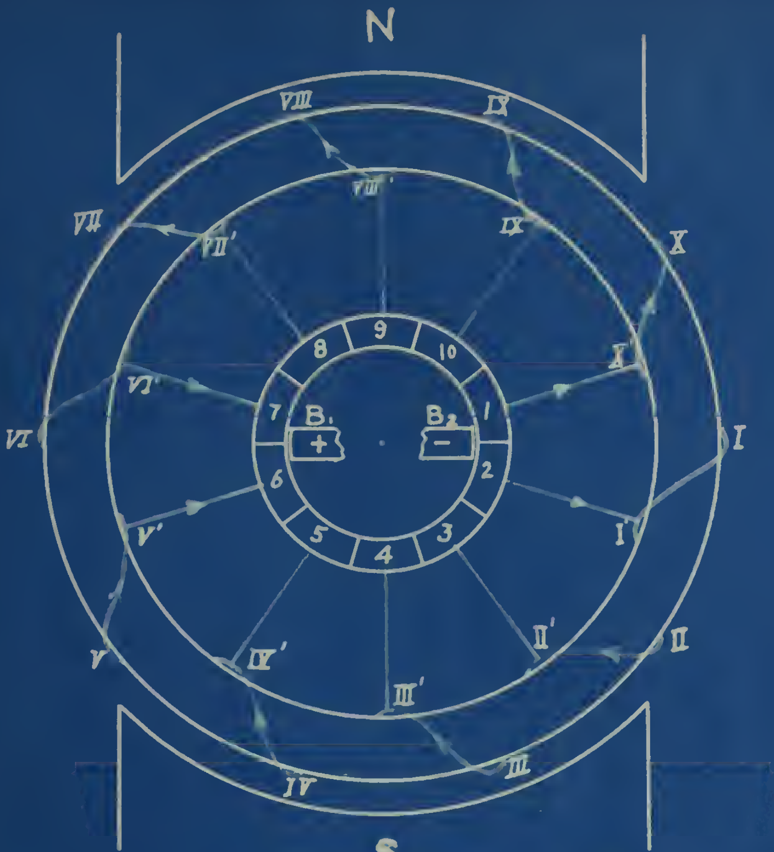

N
VIII
IX
VIII'
VII
VII'
IX'
X
9
8
10
X'
7
B1
B2
1
VI'
VI
+
−
I
6
2
V'
I'
5
4
3
II'
II
V
IV'
III'
III
IV
S

Fig. 14


N
+
−
S
S
−
+
N

Fig. 15

It is evident from figure 14 that there will be a commutator segment for every turn on the armature, this is not necessary as there may be many turns per segment, at times reaching as many as fifty or eighty, depending upon commutation conditions.

Figure 15 represents a ring armature with four poles and hence four neutral points around the armature, and for electrical balance we will need four brushes. By tracing through the circuits it is evident that we now have four paths in parallel through the armature windings. This represents the multiple circuit winding there being as many circuits through the armature as there are poles.

DRUM WINDING.

From the preceeding figures it is evident that each coil or spiral consists of two parts, i.e. the active conductor lying on the surface and the conductor lying under the ring and forming the connection between active conductors.

The essential difference between the ring and drum winding lies in the fact that in the latter both conductors are placed on the surface of the armature and thus the inactive connector becomes an active conductor, while the connector between these conductors extends along the end of the armature.

From this difference we arrive at two essential points. First, since in the ring winding there is an inner wire corresponding to each surface conductor the number of conductors on the surface of a corresponding drum armature will be twice what it was in the original ring. Consequently whether the ring were wound with an odd or even number of coils the number of conductors on the drum

wound armature becomes even.

Secondly, since in the case of the ring winding the conductor lying on the inner surface carried current in a direction opposite to the conductor to which it is directly connected, it is evident that this conductor when placed on the drum wound armature must be placed in a field of opposite polarity to the conductor to which it is directly connected.

Figures 14 and 16 show the comparison of a singly re-entrant ring winding developed into a drum winding. The conductors are so numbered as to indicate in the drum winding, the conductor and connector of the ring winding.

The Roman numbers indicate the turns, I = old conductor on ring, I' = new conductor which on the ring machine was the connector.

By looking at the figures it may be seen that the old conductors are odd numbered and the new conductors which were connectors for the ring are even as indicated by the Arabic numerals marked continuously around the armature.

The difference in the number I and I' is odd and constitutes what is known as the front pitch = y_f.

The difference between the numbers of I' and II is also odd and is called back pitch = y_b. The front and back pitch in this case are respectively 11 and 9.

$$y_f = -12 - (-1) = -11 \qquad (43)$$

$$y_b = +12 + (-3) = +9 \qquad (44)$$

$$y_f - y_b = 2 \qquad (45)$$

For lap or multiple winding, it is seen that when tracing through the windings one progresses in one direction at the front,

and the opposite at the back. One pitch is usually considered + and the other - as shown above. Front pitch is the interval between conductors connected together at the front and is usually -. Back pitch is the interval between conductors connected together at the back and is usually +. Either may be designated in number of slots, inches or degrees spanned by the coil.

Since the number of segments in this case is equal to the number of coils and since each coil constitutes only one turn, it is evident the number of segments = 1/2 the number of conductors.

COMMUTATOR PITCH.

If the connections to the commutator be numbered successively, then, since there are always as many commutator sections as coils their numbers may also be considered as applying to the coils. But the order of the ends of the armature coils may not be the same as these connections, nor need they be connected to neighboring segments. For this reason the term commutator pitch is used to denote the numerical difference between the number given to one commutator segment and the next one to which the other end of the same coil is connected. This is clearly brought out by the wave winding.

MULTIPOLAR DRUM WINDINGS.

Multipolar windings are divided into two general classes, i.e. multiple circuit or lap-wound and two circuit or wave wound. The name lap winding comes from the fact that in this type of winding the turns lap back, while in the wave winding the turns progress from pole to pole. This will be seen more clearly from the

Fig. 16

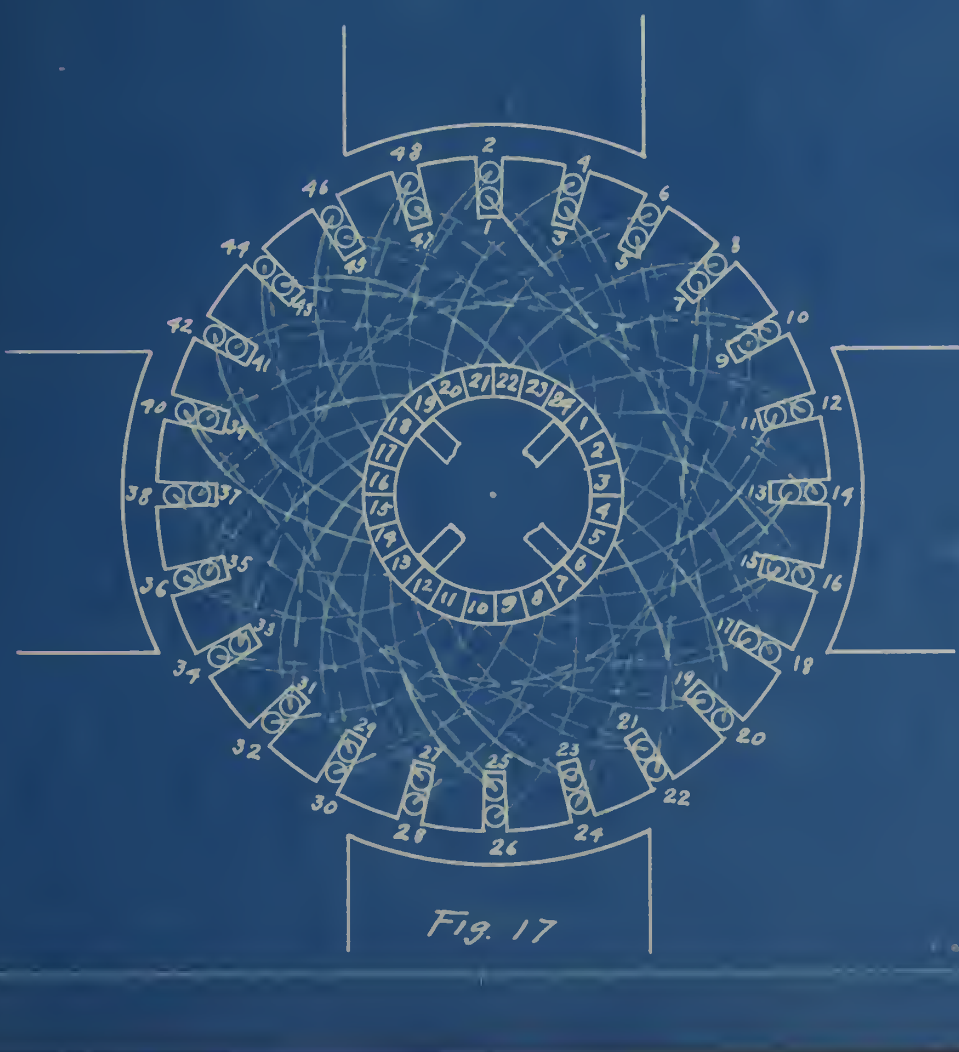


Fig. 17

diagrams that will follow.

THE SIMPLE MULTIPLE OR LAP WINDING.

It has been shown that for the two pole drum winding a conductor under a north pole is connected to a conductor which lies under a south pole, and the same holds true for a multipolar machine.

From an electrical point of view these conductors need not lie under adjacent poles, but for mechanical reasons the armature may be more easily wound and less copper is required for end connections when the coil only spans from one pole to the adjacent pole.

As far as possible those conductors lying under similar portions of adjacent poles should be connected together as this tends to give the maximum e. m. f. for a given speed and flux. If the forward pitch is made very much greater or less than the pole pitch both conductors will come under the influence of the same polarity and tend to give zero e. m. f. for this particular turn. Nevertheless windings are in use where the front pitch is less than the pole pitch. This type of winding is known as the fraction-pitch or cord winding and has the advantage of using a smaller amount of copper for end connections than does the full pitch winding. As a general rule the front pitch y_f is usually equal to or slightly less than the pole pitch and when expressed in number of teeth must be odd. The back pitch is $(y_f - 2)$ and given the + sign.

Furthermore it is evident that this winding divides itself into as many circuits as there are poles and that the e. m. f. and

hence the currents in each of these windings oppose each other and hence for the proper collection of current there must be as many brushes as there are poles and the brushes must be placed at the points where these currents meet.

Figure 18 represents a four pole lap winding, the winding formula being,

$$y_f = -6 - (-1) = -5 \quad (46)$$

$$y_b = 8 - 1 = +7 \quad (47)$$

$$\text{Commutator pitch} = 1 \quad (48)$$

$y_b = (y_f - 2)$ and given the positive sign to denote progression in the opposite direction.

There are in this winding twenty-four armature conductors, twelve coils of one turn per coil or two conductors per coil and twelve commutator segments.

It is sometimes convenient to express the connections in a conventional winding diagram. Thus for the above winding will be

1 8 3 10 5 12 7 14 9 16

In the multiple circuit winding as shown in Figure we find by tracing through the windings the following conditions or connections. Starting with conductor number 1 8 3 10 5 12 7 14 etc. That is starting with conductor number 1 we proceed to the back of the armature the by end connection across the back to conductor number 8, then to the front of the armature and through commutator segment number 2 to conductor number 3 etc. This shows the laping back of the windings from which the name "Lap Winding" was derived. Figure 19 represents the winding diagram for figure 18 developed, which makes the positions of conductors

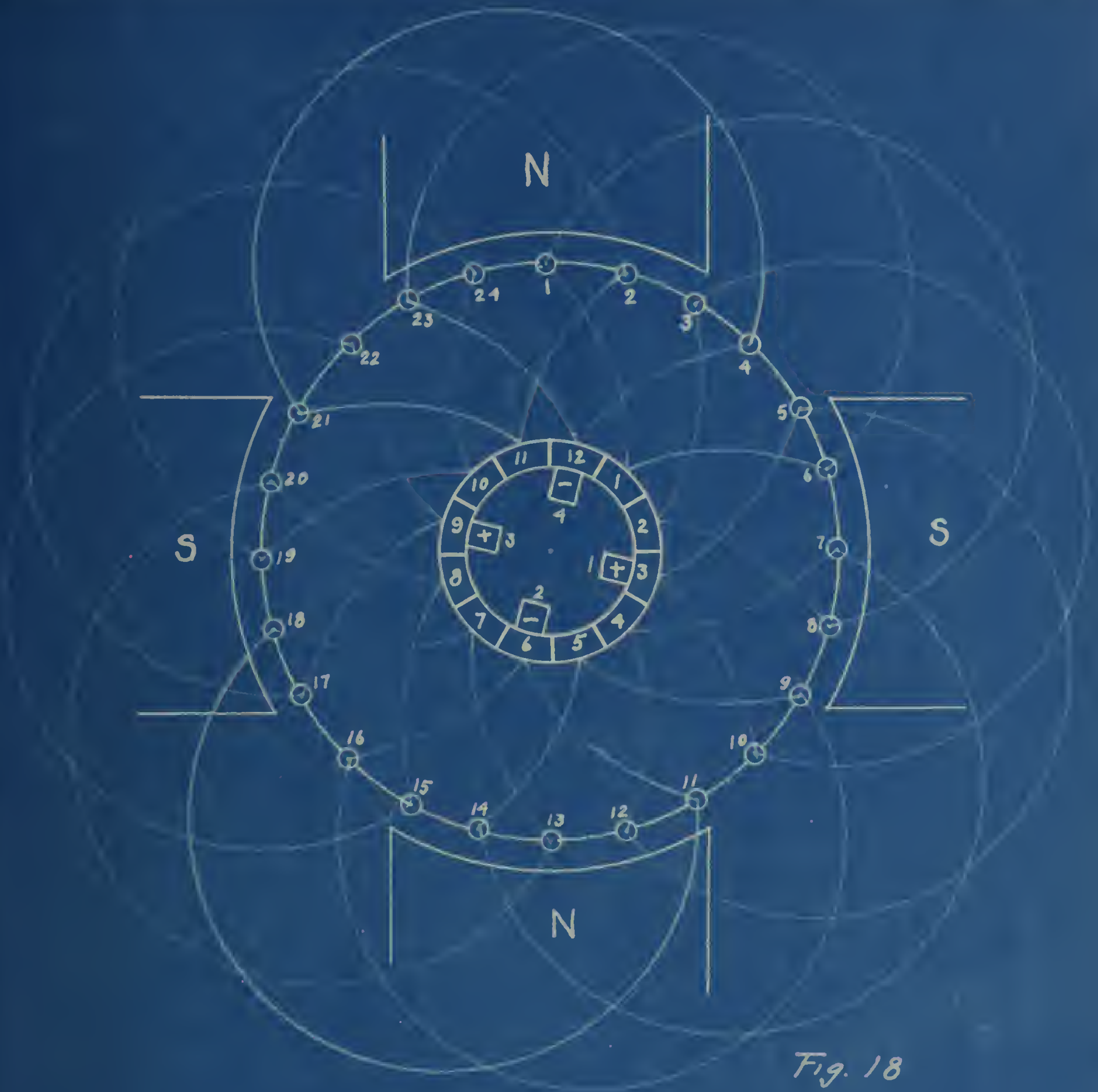


Fig. 18

Fig. 18a

relative to poles and commutator segments more easily understood.

RESISTANCE OF DRUM ARMATURE WINDING.

Let L' = length of wire in armature in inches.

a = cross section area in square inches.

ρ = specific resistance of copper.

P = the number of poles.

Then resistance of total wire in series = $\rho \dfrac{L'}{a}$ ohms.

The resistance per path = $\dfrac{\rho \frac{L'}{a}}{P}$ ohms.

The resistance of all paths in multiple = $\dfrac{\rho \frac{L'}{a}}{P.P} = \dfrac{\rho}{a} \dfrac{L'}{P^2}$ ohms.

EQUALIZING RINGS.

In multipolar machines and especially large multipolar machines it is essential that the e. m. f. in armature sections be exactly balanced, otherwise cross currents will flow between them, which increases the heating, lowers the efficiency and decreases the out put of the machine for a definite temperature rise.

Since the number of conductors per circuit is equal, the difference in potential will be due to either a difference in flux per pole or a difference in flux distribution over the pole surface, caused by wearing of the bearings, bending of the shaft, or any irregularity of pole face or teeth.

To prevent these currents from passing through all the armature conductors, they are made to circulate through what is known as equalizing rings, whereby the potentials of the different circuits are made equal.

Let us take an example of an armature with one hundred forty-four conductors and wound for an eight pole field structure. The number of conductors per pole will then be equal to eighteen and hence the pitch in conductors from say conductor number 1 and a conductor similarly situated under a similar adjacent pole will be thirty-six. Therefore conductors #1, #37, #73 and #109 occupy the same position with respect to poles of like polarity and should therefore have the same potential differences if the flux cut by each is the same and in order to equalize there potentials they should be connected to the same ring.

Now suppose we choose to use six equalizing rings. Then we must connect every sixth conductor to a ring or our next series will be connected to ring #2, 7 - 43 - 79 - 105. And to ring #3, 13 - 49 - 85 - 121, to ring #4, 19 - 55 - 91 - 127, to ring #5, 25 - 61 - 97 - 133, to ring #6, 31 - 67 - 103 - 139.

We could have used nine rings and still farther reduced the path in the armature by connecting every fourth conductor to a ring instead of every sixth conductor. Which means that the more equalizing rings used the less will be the circulating current in the armature bars.

Equalizing rings are joined with the conductors either at the commutator segments or at the rear of the machine.

TWO CIRCUIT OR WAVE WINDING.

In the wave winding there are only two paths through the armature regardless of the number of brushes used or the number of poles on the machine.

Let us trace through the simple wave winding as illustrat-

ed by figure 20 . Starting at segment #1 we proceed to conductor #1 then to the rear of the armature and by back connection to conductor #6 thence to the front end again along conductor #6 and then by front connector (not to segment #2 in the backward direction) but to segment #6 in the same direction etc., always progressing in the same rotation around the armature and never laping back as in the multiple circuit winding.

From the figure it is seen that

$$y_f = -7 - (-2) = -5$$

$$y_b = 8 - 3 = +5$$ and also the commutator pitch = +5.

In the wave winding two brush sets, a positive set and a negative set must be used and for an N pole machine N sets may be used, but two sets will always suffice. The fact that two brush sets are sufficient is shown in figures 19 and 20 .

Suppose we use first four brush sets one at each neutral point. Commutator segments #1 and #6 under the two + brushes are electrically connected through the two conductors #1 and #6 and their end connections. In the figure it is seen that these two conductors lie between the poles and are not generating any electromotive force, hence they serve only as cross connections between the two positive neutral points. The negative neutral points are similarly connected by idle conductors #5 and #10.

Therefore the only function of the added brushes is to aid the first set in collecting current from and delivering current to the commumator due to the increased contact area, but the distribution of current in the windings remains unchanged.

When, however only two brush sets are used on an N pole

armature then $\frac{N}{2}$ winding elements all in series are short circuited under a brush when the brush touches two segments (simplex winding).

Hence the wave winding using only two brush sets has a greater tendency to spark than the lap winding, but since the wave winding has only two circuits in multiple it follows that one-half the total turns on the armature are in series at all times and therefore for a given number of total turns a higher e. m. f. will be generated than in a multiple circuit winding of more than two circuits for the same flux and speed.

TWO LAYER WINDING.

Figure 17 , page 32 illustrates a multiple drum winding, but in this case slots are used and the conductors are placed in two layers, two conductors per slot. This is the common type of winding in use at the present time, except that we often find the conductors in a slot grouped into layers of several conductors each. It will be noticed in the figure that all the upper conductors are even numbered and all the lower conductors are odd, and that a conductor in the bottom of a slot under one pole is connected to a conductor in the top of a slot by the back end connection and this conductor is under the next pole.

In this case

$$y_f = -18 - (-5) = -13$$

$$y_b = 20 - 5 = 15$$

Commutator pitch = 1.


N
S
S
N
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
+
−
+
−

Fig 19

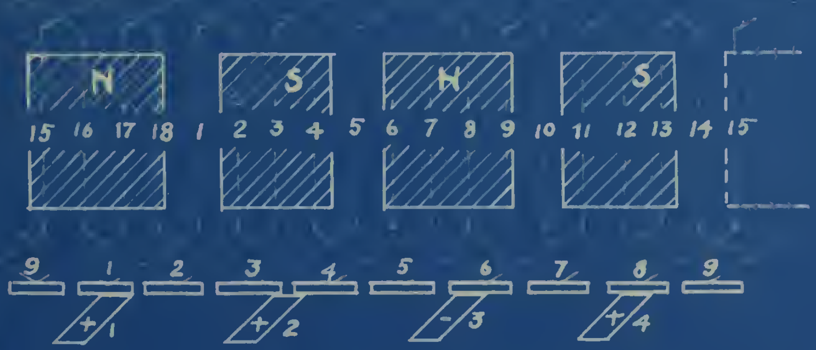

N
S
N
S
15 16 17 18 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
9 1 2 3 4 5 6 7 8 9
+ 1
+ 2
− 3
+ 4

Fig. 20

COMMUTATION.

Consider the case of coil #1, figure 21 which is undergoing commutation or is short circuited by the brush. It is evident from the figure that coil #19 is carrying current in a direction opposite to that of coil #2 and that the values of these currents are equal to the normal coil current of the armature I_p. Therefore it follows, that, since coil #1 is passing from position #19 to position #2, the current in this coil must change from a value I_p to zero and then to a value I_p in the opposite direction = ($-I_p$ with reference to coil #1) during the time of short circuit.

Figure 22 thus illustrates the reversal of current in coil #1 according to the sine law, i.e. the current changes from a value I_p at A to $-I_p$ at B, decreasing to zero and increasing again along the sine curve. Where A represents the instant of short circuit and B represents the instant the brush open circuits the coil.

Figure 23 represents the position of coil at the instant the brush makes the short circuit when the current = I_p.

Since the frequency of commutation depends upon the length of time the coil is short circuited, it is evident that the following factors will determine its value.

1. Peripheral speed of the commutator.
2. Width of the brush.
3. Thickness of insulation between segments.

Let V_c = peripheral speed of the commutator in inches per second.

Let W_b = width of the brush at commutator surface.

Fig. 21

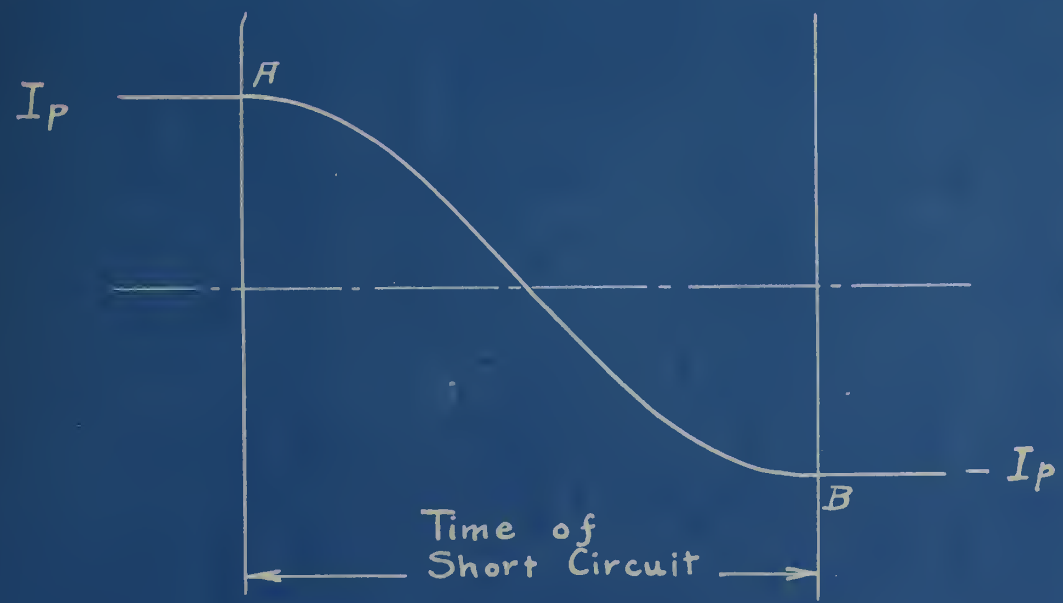


Fig. 22

Let W_i = thickness of insulation between segments.

Let f_c = frequency of commutation.

Now the time required to change the current in coil #1 from I_p to $-I_p$ is evidently the time that the coil is under the brush and this will be = T_c.

$$T_c = \frac{\text{Brush width in inches, thickness of insulation between segments.}}{\text{Peripheral speed of the commutator in inches per second.}}$$

$$T_c = \frac{W_b - W_i}{V_c} \qquad (49)$$

But since the current change only represents one half cycle, from I_p to $-I_p$, T_c = time of one half cycle and the time of one complete cycle will be

$$2T_c = \frac{2\,(W_b - W_i)}{V_c}$$ and since the frequency in cycles per second = $\frac{1}{2T_c}$

$$f_c = \frac{V_c}{2\,(W_b - W_i)} \qquad (50)$$

EFFECT OF RESISTANCE ON COMMUTATION.

In figure 24 coil #1 is carrying current in a direction as indicated by the arrow and is about to become short circuited by the brush. At the instant of short circuit as shown in figure 25 the resistance between the brush and segment #1 is very high and only a small amount of current from coil #19 will pass through coil #1 and out segment #2 into the brush. In figure 26 there should be equal resistance between segment #1 and segment #2 so that current from #19 will pass to the brush through segment #1 and current from coil #2 will pass to the brush through segment #2

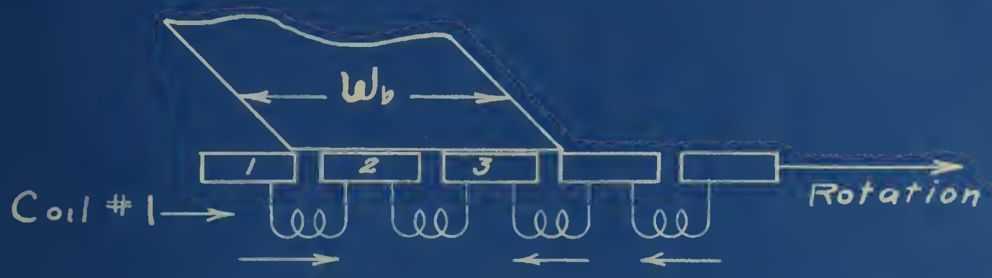


Fig. 23

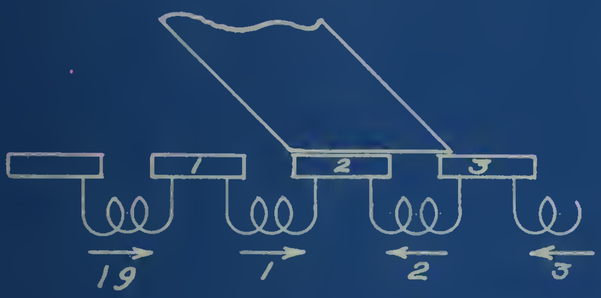


Fig 24

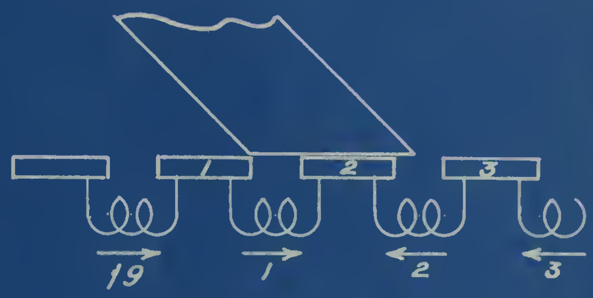


Fig. 25

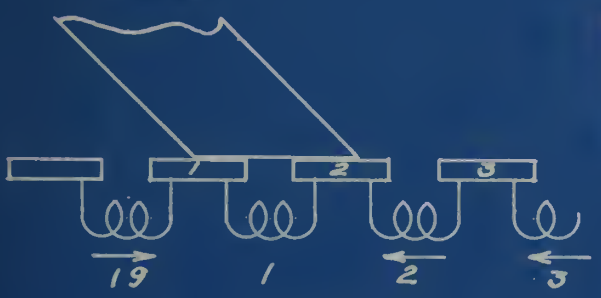


Fig. 26

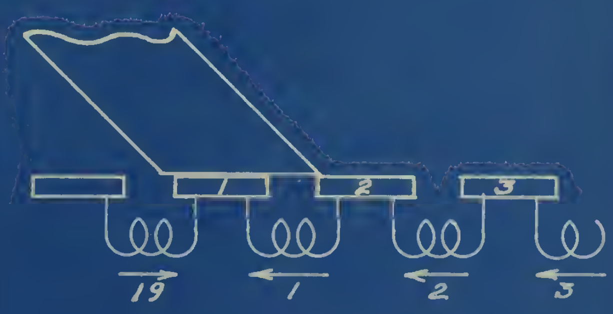


Fig. 27.

and coil #1 will have zero current. In figure 27 the resistance between segment #1 and the brush is less than between segment #2 and the brush so that current from coil #19 will leave through segment #1 and current from coil #2 will leave part through segment #2 but mostly through coil #1 and segment #1. Thus we have the current reversed in coil #1 due to the contact resistance of the brush on the commutator. This is known as resistance commutation, the brushes being high resistance carbon.

PURE E. M. F. COMMUTATION BY MEANS OF INTERPOLES.

When a coil is short circuited under the influence of the interpole there are three E. M. F.'s acting therein.

1. The e. m. f. due to the current flowing through the coil having resistance r if i = the current at any instant during short circuit, then ir = this e. m. f. and since we are assuming the commutation is to be caused by a counter e. m. f. we will neglect the varying resistance of the brush due to contact.

2. The e. m. f. due to self induction caused by the changing current in the coil itself and if L = coefficient of self induction of the coil this e. m. f. = $L \frac{di}{dt}$.

3. The e. m. f. set up in the coil due to its rotation through the magnetic field of the interpole. Let this e. m. f. = e.

Then $$ir + L \frac{di}{dt} - e = 0 \qquad (51)$$

$$\frac{di}{i - \frac{e}{r}} = - \frac{r}{L} dt. \qquad (52)$$

and

$$\int \frac{di}{i - \frac{e}{r}} = \int - \frac{r}{L}\, dt. \tag{53}$$

$$\text{Loge}\ (i - \frac{e}{r}) - \text{Loge}\ C = - \frac{rt}{L} \tag{54}$$

Where C is an integration constant and may be determined from the following known conditions.

When $t = 0$, $i = I$, where I is normal current in the conductor or the current the conductor is carrying the instant before being short circuited.

By substitution

$$\text{Loge}\ (I - \frac{e}{r}) = - \text{Loge}\ C \tag{55}$$

Or

$$C = (I - \frac{e}{r})$$

Again at the instant of open circuiting the coil $t = t_c$ if there is to be no spark, i must be equal to -I. Hence in equation 55 above we may substitute these values and determine (e).

Or

$$\text{Loge}\ (- I - \frac{e}{r}) - \text{Loge}\ (I - \frac{e}{r}) = \frac{-rtc}{L} \tag{56}$$

$$\frac{(- I - \frac{e}{r})}{(I - \frac{e}{r})} = \epsilon^{\frac{-rtc}{L}} \tag{57}$$

Or

$$e = \frac{- Ir\ (1 + \epsilon^{\frac{-rtc}{L}})}{(1 - \epsilon^{\frac{-rtc}{L}})} \tag{58}$$

Where e = e. m. f. induced in the armature coil by the inter-pole flux necessary to open circuit the coil carrying I amperes in the correct direction without drawing a spark at the brush.

In the above expression I is known and is the current per

armature conductor.

t_c = time of commutation and is easily calculated as previously illustrated.

r, is to be sure a variable quantity as has been shown by the discussion on resistance commutation, but for e. m. f. commutation the brush resistance may be made very small since we do not attempt to reverse the current by means of this resistance. Hence it is safe to use for r only the resistance of the coil under short circuit.

L = coefficient of self induction of the coil undergoing commutation.

Hobart has found that for slots whose depth is not more than 3.5 time their width, the flux set up is on an average ten C. G. S. lines per inch of slot (axial length) and two C. G. S. lines per inch of end connection per ampere turns. For ordinary drum windings the length of one armature turn will be very nearly equal to $2L + 10\ D/p$. (59)

Where L = length of conductor in iron = length of armature in inches

D = diameter of the armature in inches

P = number of poles.

The total lines produced by one ampere flowing in one turn will be $\varphi = 20\ L = 20\ \frac{D}{P}$. In practically all direct current generators there are two coil sides in each slot short circuited at the same instant, but by different brushes. Therefore the total flux produced will be double the value φ, but this flux is not all interlinked with the coil short circuited by one brush, since the coils are parallel only in the slot the end connections

passing in opposite directions at the end of the armature.

Thus

$$\varphi = (40\ L + 20\ \frac{D}{P})$$

$$= 20\ (2\ L + \frac{D}{P}) \qquad (60)$$

Now let n = number of turns per segment. Then one ampere will produce a flux

$$\varphi = 20\ (2\ L + \frac{D}{P})\ n \qquad (61)$$

and if the brush short circuits C coils at any instant the flux will be

$$\varphi = 20\ n\ C\ (2\ L + \frac{D}{P}) \text{ per ampere.} \qquad (62)$$

Now since $L = \frac{n\ \varphi}{i}$ = n x flux per ampere (63)

$$L = 20\ n^2\ C\ (2\ L + \frac{D}{P}) \qquad (64)$$

if i is in amperes then.

$$L \text{ in henrys will be } L = \frac{20\ n^2\ C}{10^8}\ (2L + \frac{D}{P}). \qquad (65)$$

The reactance voltage may now be calculated.

Since f_c = frequency of commutation and

I = coil current

$$X = 2\ \pi\ f_c\ L \qquad XI = 2\ \pi\ f_c\ L\ I$$

$$E = XI = 2\ \pi\ f\ L\ I = \text{reactance voltage.} \qquad (66)$$

This is of course on the assumption that the current dies away and increases again according to the sine law, which is not true since we know that if a coil or circuit containing resistance and inductance and carrying a current is short circuited the current at any instant after short circuit is given by the equation $i = C\ \epsilon^{\frac{-rt}{L}}$ procided r is a constant, similarly the current in an

inductive circuit takes a definite time to rise, and the current at any instant is given by

$$i = \frac{E}{r}\left(1 - \epsilon^{\frac{-rt}{L}}\right) \tag{67}$$

where $\frac{E}{r}$ represents the maximum current in the coil.

Thus if the coil has a large inductance the natural time required for the current to rise and fall is relatively large. Hence if the natural period for the current to rise and fall is greater than the time of commutation it follows that the current will tend to follow from the segment to the brush as an arc or spark, and therefore with resistance commutation it is essential that the resistance of the coil and brush be large and the inductance be small.

This result is obtained by using

(a) High resistance brushes.

(b) Deep and narrow slots.

(c) Slots that are open at the armature surface.

(d) As few turns per coil as possible.

(e) Narrow brushes.

Now if we assume sinusoidal change of current

$$X = 2\,\pi\,f\,L$$

$$X = \frac{2\,\pi\,V_c \times 20 \times N^2\,C}{2\,(W_b - W_i)\,10^8}\left(2L + \frac{D}{P}\right)$$

$$= \frac{2\,V_c\,\pi\,N^2\,C\,\left(2L + \frac{D}{P}\right)}{10^7\,(W_b - W_i)} \tag{68}$$

If we assume a straight line change of current

$$E_s = \frac{2}{\pi}\,E = \frac{4\,V_c\,N^2\,C\,\left(2L + \frac{D}{P}\right)I}{10^7\,(W_b - W_i)} \tag{69}$$

Where E and E_s are values of e. m. f. produced by self-induction in the short circuited coils. E and E_s should not be greater than four volts in any machine.

As an example take the following dimensions of an armature to calculate E.

Diameter of armature = 12"

Number of poles = 4

Turns per segment = 1

Segments short circuited = 3

Length of armature = 5"

Width of brush = .5"

Thickness of insulation = .03"

Diameter of commutator = 9"

Revolutions per minute = 1000

Then $V_c = 470$ and $f_c = 500$

Solving we find E = 2.45 volts.

E_s = 1.56 volts.

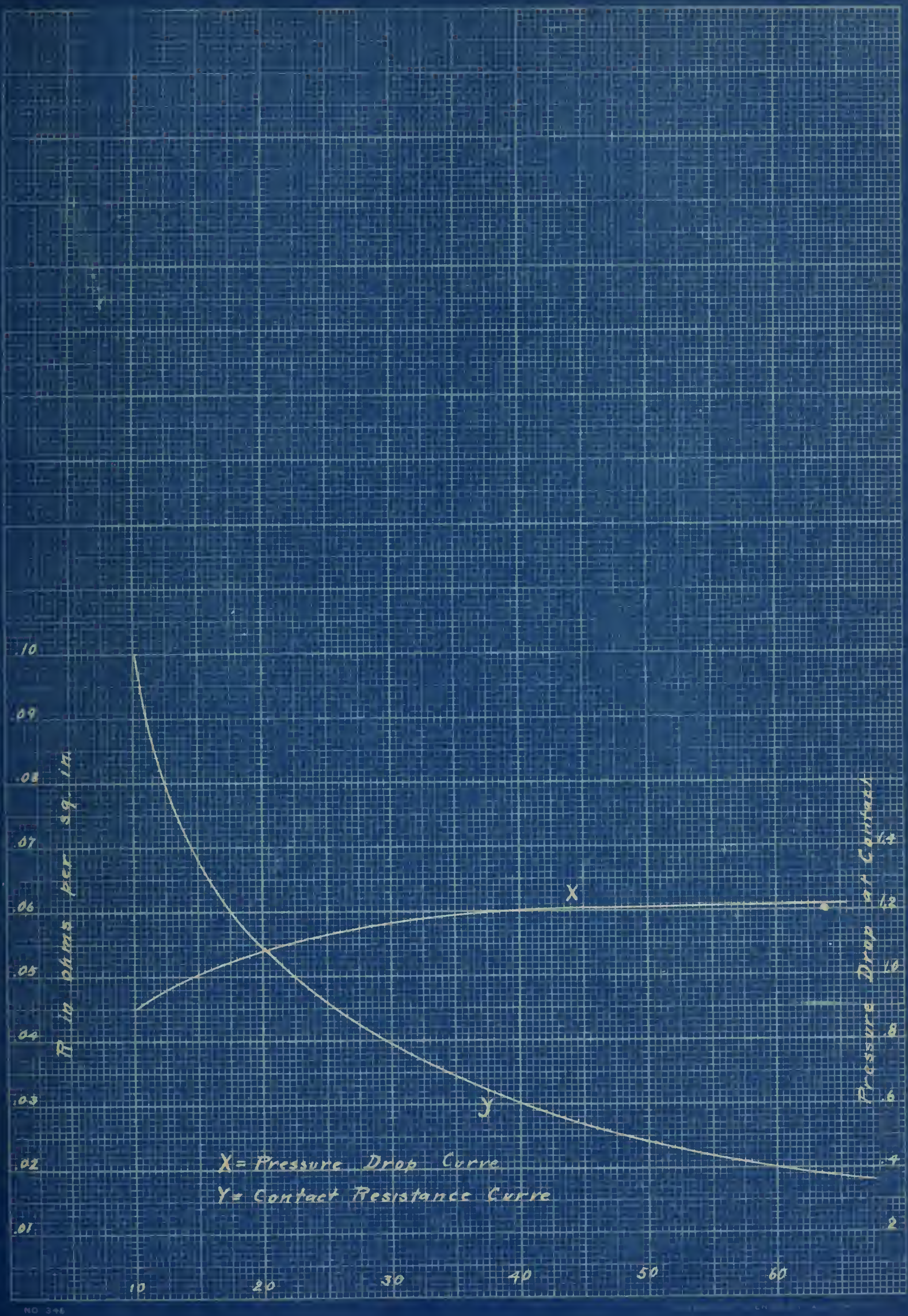
Some authors state that E_s should not be greater than 55 ÷ current density of the brush, where carbon brushes are used. Thus

$$E_s = \frac{55}{\text{current density}}$$ and for most types of machines E_s would then be about

$$E = -\frac{55}{30} = 1.83 \text{ volts} \tag{70}$$

Figure 27a page 47 shows the variation of resistance and pressure drop with current density for a hard carbon, the pressure being one and one-half pounds per square inch.

The following table gives values for several standard



.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
R in ohms per sq. in.
10
20
30
40
50
60
X
Y
1.4
1.2
1.0
.8
.6
.4
.2
Pressure Drop at Contact
X = Pressure Drop Curve
Y = Contact Resistance Curve
NO. 346

brushes with the makers estimate of proper density and pressure to be used, also values of E_s and coefficient of friction for each.

Brush	Current Density amperes per sq. in.	Coefficient of friction	Pressure lb. per sq. in.	E_s
A	40	.22	1.5	1.4
B	45	.22	2.0	1.2
C	30	.28	1.5	1.8
D	35	.25	1.5	1.5
E	28	.28	1.5	1.9
F	25	.30	1.5	2.0
G	55	.22	2	1.0

A brush should always be made wide enough to span at least one segment and in practice the average brush spans about three segments, the average brush thickness is 3/8 to 3/4 of an inch. There should always be two brushes per arm so that an accident to one brush will not put the machine out of operation. Too great an axial length per brush means poor contact an axial length of one inch to one and one half inches is common practice.

Example of commutator.

200 K.W., 500 volts, 400 R. P. M., 6 pole lap-wound without interpoles.

Diameter of armature	35"
Diameter of commutator	30"
Peripheral speed	2940"
Electrical loss	960 watts
Coefficient of friction	.3
Friction loss	800 watts

Total losses	1760
Cylindrical surface	700 sq. in.
Length of face	8"
Number of segments possible	$\frac{\pi \times 28}{.18} = 488$
Width brush	3/4"
Amperes per arm	133
Brush area per arm	4.4 sq. in. (30 amp. sq. in.)
Axial length	5.9 inches
Axial length for 6 brushes	8 inches
Watts per sq. in.	2.5

Thus in this case the length is determined by the brushes rather than the temperature rise since a temperature rise based on 3 to 4 watts per square inch would have been satisfactory.

COMMUTATOR.

When commutators are driven at high speeds there usually arises great trouble due to chattering of the brushes and also if the frequency of commutation is increased due to the high speed there may be a large reactance voltage set up which is opposed to sparkless commutation as described under commutation.

Commutators are at times run at peripheral speeds exceeding 5000 feet per minute but a more conservative value would be about 2500 feet per minute at which speed good operation is assured. In choosing the number of commutator segments use may be made of figure 28.

Using the value of 2500 feet per minute for commutator speed and knowing the speed of rotation we may at once determine

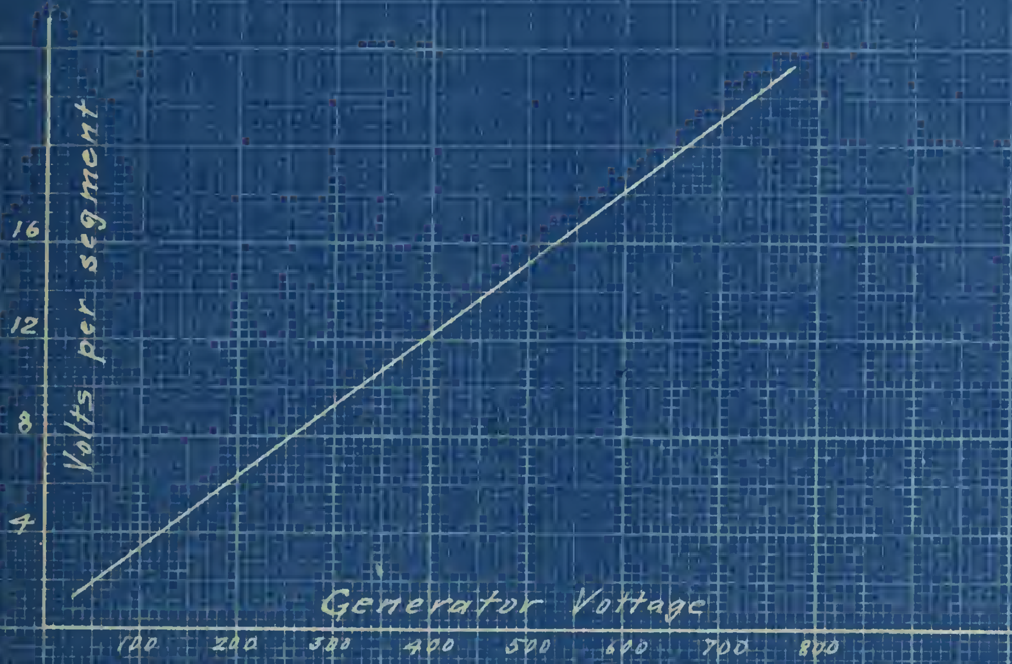

Volts per segment
16
12
8
4
Generator Voltage
100
200
300
400
500
600
700
800

Fig 28

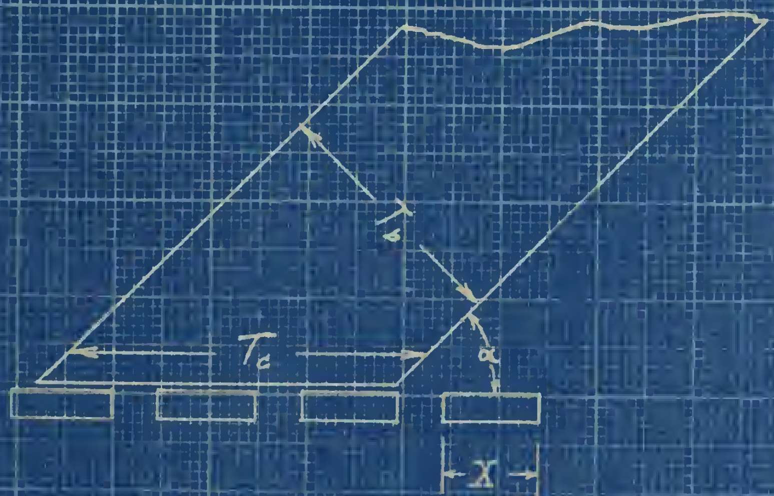

T_c
X

Fig. 29

circumference of commutator from

$$V_c = \Pi \times D \times R.\ P.\ M. \tag{71}$$

Mica or micanite is usually used to insulate between commutator segments and its thickness varies from 20 to 40 mils. Thus we choose a value as say 30 mil = .03" and knowing the diameter of commutator and number of segments it is an easy matter to calculate the width of each segment at the surface of the commutator.

Since if we let X = width of one segment in inches and .03" thickness of insulation and Z = number of segments then

Z (X + .03) = the circumference of commutator, and circumference in feet x R. P. M. = peripheral speed in feet per minute = 2500, we have

$$\frac{Z_1}{12} (X'' + .03'') \times R.\ P.\ M. = 2500$$

from which we may solve for X since Z is known. Commutator segments are usually made from .18 inches to .40 inches in width at the commutator surface. The length of the commutator depends first on current to be taken off at brushes arm, the number of segments spanned by the brush each and the current density, the length also must be such as to give the proper radiation surface. It is usual to make the brush span approximately .3 segments, and using the above notation the thickness of the brush will be = T_b on commutator. Let T_c = thickness of brush resting.

$$T_c = 3X + .03 \times 2 \tag{72}$$

$= (3X + .06)$ and if the brush was placed perpendicular to the commutator surface the thickness of each brush would be = T_c = (3X + .06). But since the brushes are usually

set at an angle α = approximately 45 degrees, the thickness of the brush T_b will be

$$T_c \sin\alpha = T_b \qquad (73)$$

and for α = 60 degrees. See figures 29 and 30.

$$T_b = .866\ T_c$$

The cross section area of brush will be T_b x L_b and this must be such a value as to carry the total positive arm current at a density equal to 30 amperes per square inch.

Or

I = total current.

Y = number of positive brushes.

$\frac{I}{Y}$ = current per positive brush.

Area of each positive and negative brush = $\frac{I}{30\ Y}$

$$\frac{I}{30\ Y} = T_b \times L_b$$

or

$$L = \frac{I}{30\ Y\ T_b} \qquad (74)$$

It often happens that if the total current per positive brush arm were to be taken through one carbon brush its width L_b would be great in comparison to its thickness T_b and it is customary to make L_b = 1 inch to 1 1/2 inches and thus collect the current per brush arm with several small brushes, rather than one large brush. This gives the added advantage, i.e. greater area of commutator, greater radiating surface.

Since the thickness of insulation between commutator segments is usually kept constant, it is evident that the width of the segment at its inside diameter must be less than at the surface

and consequently the commutator bar must be tapered. The taper depending upon the diameter of the commutator and the depth of segment.

The useful depth of a commutator segment is represented by D and is that amount which the commutator can be worn or turned down and still be kept in service. This is a varying quantity for different sizes of machines. See figure 31.

COMMUTATOR LOSSES.

1. Electrical losses are due to the commutator resistance loss in bars and contact between bars and brushes. The contact resistance for carbon brushes has been found to be about .03 ohm per square inch of contact at a pressure of 1.5 pounds per square inch. Hence the loss due to contact may be found by,

$$\text{Current per brush arm} = \frac{\text{total current}}{\text{number of positive or negative arms}}$$

$$\text{Area of brush surface per arm} = \frac{\text{current per arm}}{\text{current density}}$$

$$\text{Resistance of contact per arm} = \frac{.03}{\text{area arm}} \qquad (75)$$

Loss at each brush arm = $(\text{current per arm})^2$ x resistance per arm.

Total loss = number of arms x (positive and negative arms) x loss per arm.

Total current x current density x .06 if current density = 30, loss = total current x .18. (76)

Let total current = I

Let number of positive brushes = Y

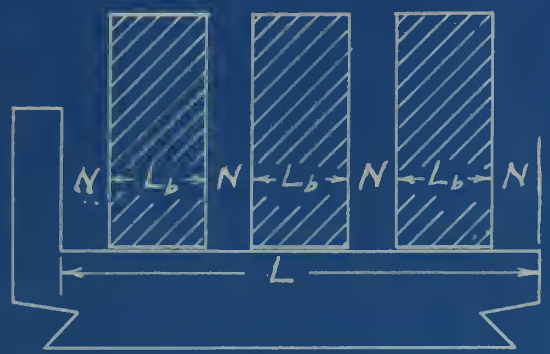


Fig. 30

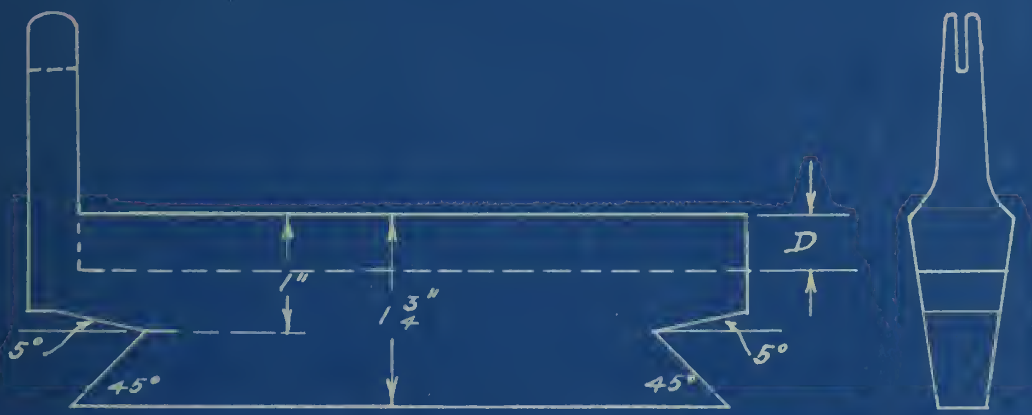


Fig. 31

Let number of positive and negative brushes = 2 Y

Current density = 30 amperes per square inch

Resistance per square inch = .03 ohms

Current per positive brush = $\frac{I}{Y}$

Then brush area in one positive arm = $\frac{I}{30\ Y}$ (77)

Resistance per brush = $\frac{.03}{area} = \frac{.03 \times .30\ Y}{I} = \frac{.9\ Y}{I}$ (78)

Loss per brush arm = (current per arm)^2 x $\frac{.9\ Y}{I}$ (79)

$$= \frac{I^2}{Y^2} \times \frac{.9\ Y}{I} \quad (80)$$

$$= \frac{.9\ I}{Y}$$

Total loss in watts = $\frac{.9\ I}{Y}$ x 2 Y = 1.8 I. (81)

2. Mechanical = firction losses.

Coefficient of friction = .28

Brush pressure = 1.5 pounds square inch

Total area = $\frac{\text{total current}}{\text{current density}} \times 2$

Horse power = $\frac{\pi\ D\ \text{Pull} \times \text{R.P.M.}}{33000}$

Then H.P. = $\frac{\pi \times D \times \text{R.P.M.} \times \frac{\text{total current}}{\text{current density}} \times 2 \times 1.5 \times .28}{33000}$

Since one H.P. = 746 watts the watts lost will be,

Watts lost = $\frac{\pi\ D \times \text{R.P.M.} \times \frac{\text{total current}}{\text{current density}} \times 2 \times 1.5 \times .28 \times 746}{33000}$

But π D x R.P.M. = peripheral speed in feet per minute = α

The current density in the brushes should be about 30 amperes per square inch.

Let K_1= coefficient of friction.

Then watts lost = α x total current x K_1 x .00226 (82)

Total losses = total current (α K_1 .00226 + 1.8) (83)

The commutator losses should not exceed 4 watts per square inch cylindrical surface and a better value is 3 watts per square inch for a temperature rise of 40 degrees C.

Hence with the above losses known the area may at once be fixed provided we assume some peripheral speed to start with. This value seldom exceeds 3000 feet per minute and a better value would be 2500 feet per minute.

LOSSES

1. The armature $I^2_a R_a$ loss can be calculated for any load once the armature resistance is known.

2. The shunt field loss may be considered constant and equal to $I^2_f R_f$.

3. The brush friction and contact losses may be found from equation

4. The series field loss is found by equation $I_a R_s$ where R_s is the resistance of the series field and I_a = the armature current corresponding to the load considered.

5. The friction and windage losses are considered as constant for all loads and since there is no known method of calculating these losses they are usually taken as a percent of the K. W. capacity of the machine.

6. The hysteresis and eddy current losses may be found in

the following manner.

(a) Calculate the volume and weight of the teeth on armature.

(b) Calculate the volume and weight of the armature body. Knowing the density in these volumes the loss in watts per pound can be found from figure 32. Since for any frequency the loss will be the value taken from the curve times the frequency in question divided by 60.

The sumation of the above losses will give the total losses for any desired load on the machine and the corresponding efficiency can be determined by the following equation.

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{losses}} .$$

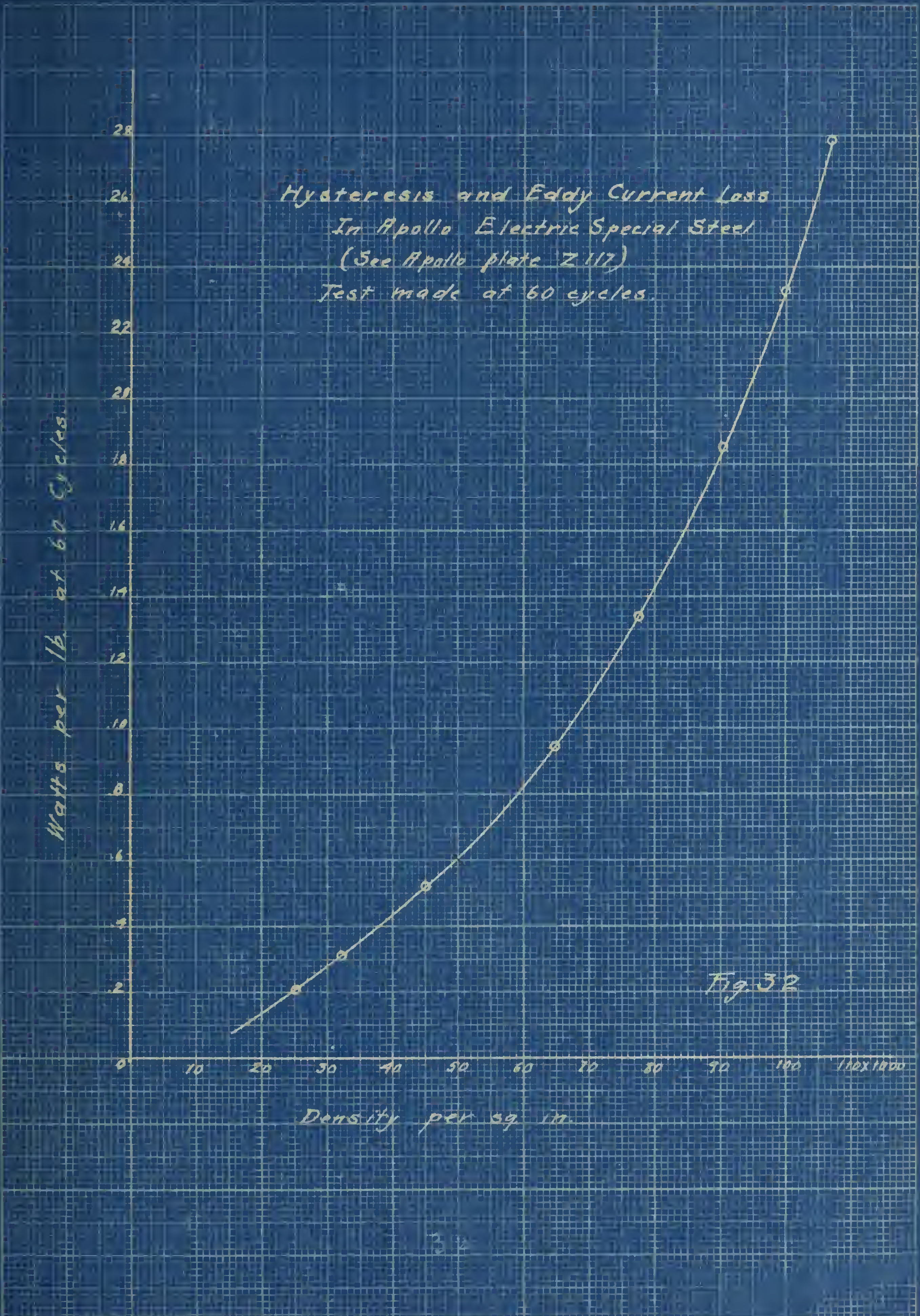

Hysteresis and Eddy Current Loss
In Apollo Electric Special Steel
(See Apollo plate Z 117)
Test made at 60 cycles.
Watts per lb. at 60 Cycles.
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
10
20
30
40
50
60
70
80
90
100
110 x 1000
Density per sq. in.

Fig. 32

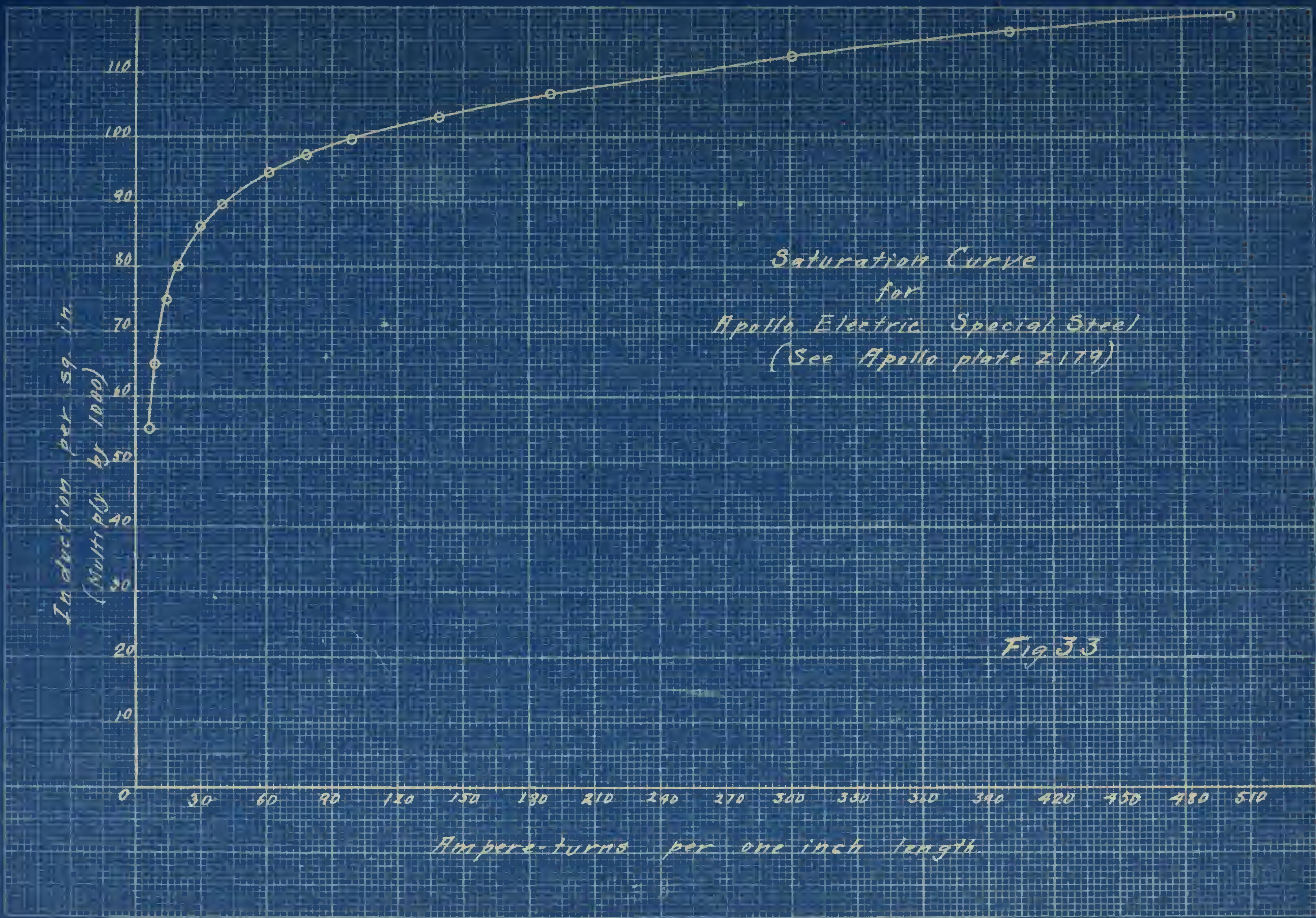

Saturation Curve
for
Apollo Electric Special Steel
(See Apollo plate Z179)
Induction per sq. in
(Multiply by 1000)
0
10
20
30
40
50
60
70
80
90
100
110
30
60
90
120
150
180
210
240
270
300
330
310
340
420
450
480
510
Ampere-turns per one inch length
NO 346

Fig 33

DIRECT CURRENT MACHINE DESIGN.

7.5 K. W., 110 v. no load, 125 v. full load, 4 poles, 1200 R.P.M.

Allow perf. speed = 2700 feet per minute.

Diam. arm = $\frac{2700}{200\pi}$ = .716 feet = 8.6 inches.

Total current = $\frac{7500}{125}$ = 60 amperes.

Current per path = $\frac{60}{4}$ = 15 amperes.

Circumference of arm = π x 8.6 = 27 inches.

Use 200 ampere conductors per inch of perf.

Amp. cond. required = 27 x 200 = 5400.

No conductors required = $\frac{5400}{15}$ = 360.

$$E = \frac{\phi\ z\ n\ P}{P^1 \times 10^8} \quad \text{or} \quad \phi = \frac{110 \times 10^8}{360 \times 20} = 1{,}530{,}000 \text{ lines.}$$

Use 8 conductors per slot.

Number slots required = $\frac{360}{8}$ = 45.

Pitch of slots = $\frac{27}{45}$ = .6 inch.

Allow 2000 amperes per square inch of conductor.

Therefore $\frac{15}{2000}$ = .0075 square inch required.

From tables use number 10 wire D. D. C.

Bare diameter = .102 inches.

Covered diameter = .114 inches.

Thickness of slot = .114 x 2 + 2 x .04 + .002 = .31 inches.

.04 is for tape and .002 for clearance.

Depth of slot = .114 x 4 + 4 x .04 + .034 = .65 inches.

Thickness of tooth = .6 - .31 = .29 inches.

Let pole arc = .76 of 1/4 the arm circumference.

Therefore no teeth under a pole = $\frac{45}{4}$ x .76 = 8.55.

Area tooth = .29K where K is the net length of arm.

Area teeth under a pole = .29K x 8.55 = 2.475K.

Allow density in arm = 120,000 lines per square inch.

Area required = $\frac{1530000}{120000}$ = 12.75 square inch.

12.75 = 2.475K or K = 5.15 inches.

Length of pole arc = $\frac{27}{4}$ x .76 = 5.13 inches.

Diameter at bottom of teeth = 8.6 - .65 x 2 = 7.3 inches.

Circumference bottom tooth = π x 7.3 = 22.91 inches.

Pitch at bottom of teeth = $\frac{22.91}{45}$ = .51 inches.

Width of slot = .31 inch.

Therefore width of tooth at bottom of slot = .51 - .31 = .2 inch.

Assume density at bottom of teeth = 120,000.

Therefore density at surface of arm. = $\frac{.2}{.29}$ x 120,000 = 82,800 lines per square inch.

$\frac{\frac{\Phi}{2}}{60000}$ = K x where x is the radial depth of the armature.

$\frac{\frac{1530000}{2}}{60000}$ = 5.15 x   x = 2.48 inches.

Ampere turns per pole = ampere turns on the armature under 1 pole.

5400 amp. cond. on armature.

Therefore 2700 ampere turns on armature.

$\frac{2700}{4}$ x .76 = .513 ampere turns under a pole.

Total length of armature = $\frac{5.15}{.9}$ + .5 for slot = 6.22 inches.

Area pole shoe = 6.22 x 5.13 = 31.9 square inches.

Φ per pole = 1,530,000.

B for pole shoe = $\frac{1530000}{31.9}$ = 48,000 lines per square inch.

B at surface of arm = 82,800 lines per square inch.

Therefore B for air gap = $\frac{82,800 + 48,000}{3}$ = 43,600 lines per square inch.

Ampere turns = .313 B_a l_a for air gap.

513 = .313 x 43600 l_a l_a = .0376 inches.

This value is too small for mechanical reasons so an air gap of 1/8 inch will be used.

Ampere turns for this length of gap is A T = .313 x 43600 x .125= 1705 ampere turns.

Length of armature coils.

$L = 2\ L_a + 10\ \frac{D_a}{P_1} + 2 \times 6.22 \times 10 \times \frac{8.6}{4} = 33.94$ inches.

Total length of wire = $\frac{z}{2}\ (2\ l + 10\ \frac{D}{p}) = \frac{360}{2} \times 33.94 = 6105$ inches = 509 feet.

$$\text{Resistance} = \frac{\frac{z}{2}(2\ l + 10\frac{D}{P}) \times \frac{R}{1000}}{P^2}$$

$\frac{R}{1000} = 1.1612$ for number 10 wire.

Resistance of armature = $\frac{509 \times 1.1612}{4^2 \times 1000}$ = .037 ohms.

$I_a^2\ R_a = \overline{60}^2 \times .037 = 133$ watts.

$\frac{60}{30}$ = 2 square inches of brush required = plus 2 x 2 - 4 square = total brush area.

Watts lost due to friction = $\frac{V_{10} x\ y \times 1.5 \times .28 \times 746}{33000}$.

Diameter of commutator,

Allow 5 volts per commutator segment.

Therefore $\frac{125}{5}$ = 25 com. seg. required per path.

Total segments required = 25 x 4 or about 100.

Number turns = 180.

Therefore use 90 commutator segments.

Commutator seg. = .2 inch wide

90 x .2 = 18 inches.

Insulation = .03 inches.

90 x .03 = 2.7 inches.

Circumference of com. = 18 + 2.7 = 20.7 inches.

Diameter com. = $\frac{20.7}{\pi}$ = 6.6 inches.

Perf. speed of commutator = $\frac{20.7}{12}$ x 1200 = 2070 feet per minute.

Therefore watts lost due to friction = $\frac{2070 \times 4 \times 1.5 \times .28 \times 746}{33000}$

= 78.8 watts.

Watts lost due to contact resistance = 1.8 I = 1.8 x 60 = 108 watts.

Thickness of brush = 2.75 x .2 + 2 x .03 = .61 inch.

Width of brush = $\frac{1}{.61}$ = 1.64 inches.

Let thickness of brush = 9/16 inch = .563 inches.

Therefore length of brush = $\frac{1}{.563}$ = 1.78 = 1 3/4 inches.

Make two brushes each 7/8 inch wide.

Surface of commutator = .2 x 3 x 90 = 54 square inches.

Armature loss = 108 + 78.8 = 186.6 watts.

Watts lost per square inch of com. surface = $\frac{186.6}{54}$ = 3.46.

Ampere turns for air gap = 1705.

A T for field = approximately 1705 x 1.2 = 2045.

Assume field current of 2 amperes.

Therefore number turns = $\frac{2045}{2}$ = 1023.

Area conductor = $\frac{2}{1000}$ = .002 square inches.

From tables use number 16 wire.

Diameter bare = .05082 inches covered D.C.C. = .062 inches.

Number layers in field coils = $\frac{1.5}{.062}$ = 24.

Thickness of coils = .062 x 24 = 1.487 inches.

Number turns per layer = $\frac{1023}{24}$ = 43.

Length of field coils = 43 x .062 = 2.665 inches.

Therefore length of magnetic core = 2.665 + 1.335 = 4 inches.

$$\text{Area core} = \frac{1530000 \times 1.2}{90{,}000} = 20.4 \text{ square inches.}$$

Make core 4 inches by 5.1 inches.

$$\text{Area yoke} = \frac{1530000 \times 1.2}{75000 \times 2} = 12.25 \text{ square inches.}$$

Let width of yoke parallel to shaft = 6.25 inches.

$$\text{Then thickness of yoke} = \frac{12.25}{6.25} = 1.96 \text{ inches.}$$

Inside radius of yoke = 4.3 + .125 + .5 + 4 = 8.925 inches.

Inside diameter of yoke = 8.925 x 2 = 17.85 inches.

Outside radius of yoke = 8.925 + 1.96 = 10.885.

Outside diameter of yoke = 2 x 10.885 = 21.77 inches.

TO CALCULATE THE AREA OF AIR GAP.

$$\text{Area teeth under a pole} = \frac{45 \times .29 \times 5.15 \times .76}{4} = 12.78 \text{ sq. inches.}$$

Pole arc + air gap length x constant.

$$\frac{\text{Distance between pole shoes}}{\text{Length of air gap}} = \frac{1.875}{.125} = 15.$$

Constant for this ratio = 2.9175.

Effective pole arc = 5.13 + 2.9175 x .125 = 5.495 inches.

Effective area of pole shoe = 5.495 x 6.22 = 34.15 sq. inches.

$$\text{Area of air gap} = \frac{34.15 + 12.78}{2} = 23.465 \text{ sq. inches.}$$

ARMATURE

Diameter at face	8.6"
Periphery	27.0"
Pole pitch at armature face	0.76"
Length between core heads	6.22"
Diameter x by core length ÷ kilowatts	7.05
Thickness of core sheets	0.02"
Number of ventilating ducts	1.0
Width of each duct	0.5"
Effective length of core (net iron length)	5.15"
Radial depth core body	2.48"
Internal diameter core	3.64"
Number of slots	45
Depth of slot	0.65"
Width of slot	0.31"
Width of slot at armature face	0.31"
Minimum width of tooth	0.20"
Width of tooth at armature face	0.29"
Total number of face conductors	360
Number of circuits	4
Style of winding	Lap
No. of conductors in series between + and -	180
Size or section of conductor (bare)	0
Size or section of conductor insulated	0.114"
Mean length of conductor per turn	340"
Pitch of winding (front and back), inches	7.2"

Pitch of winding number of teeth	12
Arrangement of conductors in slot	2 x 4
Copper section ÷ slot section (space factor)	0.322
Length of active conductor per volt	1.1 ft.
Total insulation between conductor and core	0.046
Peripheral speed (feet per minute)	2700
Centrifugal force per pound at armature face	2830 ft. lbs.

GAP

Length at centre from iron to iron	0.125"
Average length from iron to iron	0.125"
Diameter of bore of field	8.85"

POLE PIECE

Length parallel to shaft	6.22"
Length of pole arc	5.13"
Average pole arc ÷ pole pitch	6.75
Area of pole face	31.9 sq. in.

MAGNET CORE

Length radially	4.0"
Length parallel to shaft	5.1"
Width or diameter	4.0"
Area of section of core	20.4 sq. in.
Minimum distance between cores	3.5"

BOBBIN

Size of shunt wire	No. 16
Sectional area of wire	.002 sq. in.
Mean length of one turn	24.15"
Compound conductor, size or section of	.02 sq. in.

YOKE

Outside diameter	21.77"
Inside diameter	17.85"
Thickness	1.96"
Length parallel to shaft	6.25"
Area of section	12.25 sq. in.

COMMUTATOR

Diameter	6.6"
Length of segment over all	3.375"
Area of cylindrical surface	62.1 sq. in.
Active length of segment	3.0"
Number of segments	90
Width of segments	.2"
Useful depth of segment	0.5"
Thickness of insulation between segments	.03"
Peripheral speed, ft. per minute	2070
Centrifugal force per pound at face	2165 ft. lbs.

COMMUTATOR BRUSHES

Number of sets	4

Number in one set	2
Length side by side	1.75"
Width (peripheral) of brush in inches	.563"
Number of segments covered	2.5
Size of contact face (sq. inches)	1.0 sq. in.
Total area of contact, one polarity	2.0 sq. in.

ELECTRICAL

Armature

E. M. F. per circuit, no load	110.0
E. M. F. per circuit, full load	125.0
Type of winding	Lap
Number of turns per commutator segment	2
Winding formula	---
Amperes per circuit of winding	15
Amperes per sq. inch in conductor	200
Resistance of commutator risers	---
C R drop (lost volts) in armature	4.625
Ohms brush to brush (at 60 C.)	.148
Amperes per sq. inch of active peripheral belt	---

COMMUTATOR

Average volts between segments	10
Reversal density ÷ pole face density	---
Frequency of commutation	736
Amperes per sq. inch of brush contact	30.0
Contact resistance, ohms per sq. inch	.48

C R drop (lost volts) due to brush contact	7.2

FIELD COILS

Type	Form wound
Number of turns in series per bobbin	1023
Number of bobbins in series	----
Mean length of one turn	24.15"
Total resistance (at 60 C.)	9.22 ω
Amperes, full load	2.78
Amperes, no load (shunt excitation)	2.36
Amperes, per sq. inch, full load	1000
Rheostat resistance	37.6
C R drop in rheostat	88.25

COMPOUND WINDING

Arrangement of	Series
Number of turns in series per bobbin	15
Mean length of one turn	23.72"
Total resistance (at 60 C.)	.0875 ω
Amperes, full load	60
Amperes, per sq. inch, full load	1500
C R drop (lost volts) in series coil	5.25

REACTIONS

Armature

Ampere conductors, per pole	1350
Ampere conductors beneath pole	1026
Ampere conductors between poles	324

Ampere conductors per inch periph.,full load	200
Ampere turns, gap and teeth ÷ beneath pole	513
Density in gap under backward horn, 1 1/4 load	---

FIELD, PER POLE

Ampere turns, no load, no load volts	3269
Ampere turns, no load, full load volts	4202.4
% added for armature reaction	11.4
Total ampere turns, full load	4526.4
Ampere turns shunt coil, full load	3715.0
Ampere turns compound coil, full load	811.4
Ampere turns total coil, full load	-----
Inherent regulation %	-----

SCHEDULE OF CALCULATIONS FOR WINDING OF FIELD MAGNETS

At no-load

	Material	Flux from one pole (megalines)	Sectional area sq.in.	Flux density
Armature body	Steel	.7 x 65	12.77	60000
Teeth	Steel	1.53	12.76	120000
Gap	Air	1.53	23.46	65300
Pole core	Steel	1.53 x 1.2	20.40	90000
Yoke	Steel	.919	12.25	75000

At full-load

Armature body	Steel	.8695	12.77	68000
Teeth	Steel	1.7400	12.76	136300
Gap	Air	1.7400	23.46	74200
Pole core	Steel	2.0860	20.40	102200
Yoke	Steel	1.0430	12.25	85200

SCHEDULE OF CALCULATIONS FOR WINDING OF FIELD MAGNETS

At no-load

	Material	Mean Magnetic Length	Ampere-turns per unit length (from curve)	Ampere-turns needed
Armature body	Steel	5.500	5.71	31.4
Teeth	Steel	0.650	396.	257.0
Gap	Air	0.125	20420.	2555.0
Pole core	Steel	4.000	32.4	129.6
Yoke	Steel	17.000	17.13	296.0

Total ampere-turns needed at no-load = 3269.0

At full-load

Armature body	Steel	5.50	7.93	43.6
Teeth	Steel	0.65	1015.	660.0
Gap	Air	0.125	23200.	2905.0
Pole core	Steel	4.00	62.2	248.8
Yoke	Steel	17.00	20.3	345.0

Total ampere-turns needed at full load = 4202.4

Add ampere-turns needed to compensate reaction = 4526.4

Deduct amper-turns at full load = 4526.4

Difference = 811.4

Flux in armature, from 1 pole at no-load .765 lines; Flux at full-load 18695 lines. Co-efficient of allowance for dispersion at no-load 1.25; at full-load 1.25.

GAP CO-EFFICIENTS

N.B.--In calculating air-gaps the number of ampere-turns needed, per inch length across air-gap, is obtained by multiplying by 0.3313 the flux density if expressed in lines per square inch: or the number of ampere-turns needed, per centimetre length of gap, is botained by multiplying by 0.796 the flux density if expressed in lines per square centimetre. 3715.0 = ampere-turns of shunt winding. Shunt current, taken at 100 per cent of full-load current 2.78 amperes; hence Number of shunt-turns per pole = 1335.

COMPENSATING REACTION

The number of demagnetizing ampere-turns per pole may be approximately calculated by reckoning the number of conductors in the region between two adjacent pole-horns and multiplying this number by the number of amperes carried by each conductor. 324 = ampere-turns to be provided by compound winding. Full-load current is 60 amperes; hence Number of Series turns per pole = 15.

REMARKS

Add 25% to series turns.

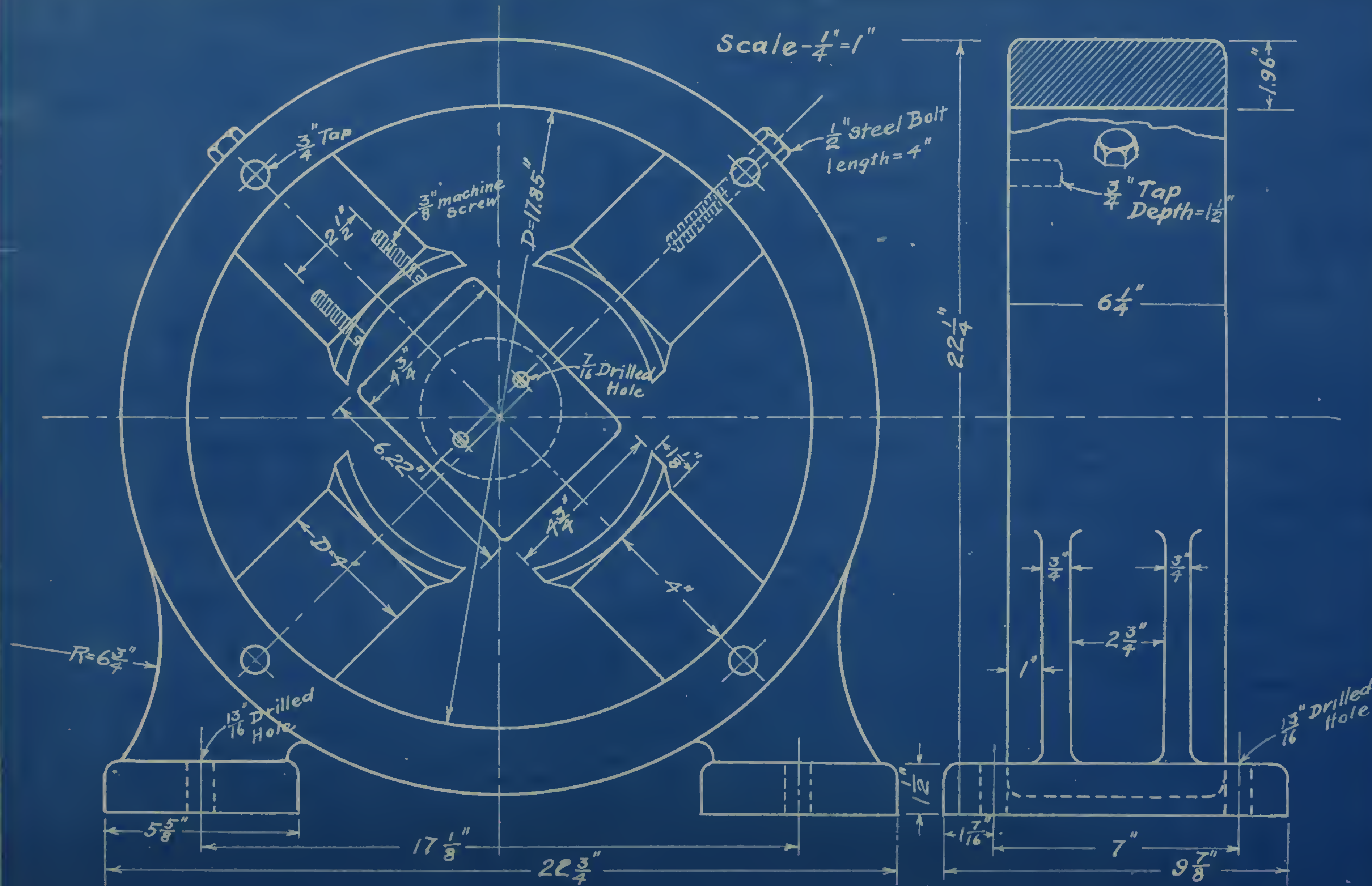
CALCULATION OF SHUNT FIELD

Assuming suitable current density 1000 amperes per square inch of copper, the appropriate wire will be number 16 B & S. This wire has .05 diameter bare, or .062 diameter covered. The number of shunt-turns being 1335 these will occupy 5.13 square inches

of winding space on the bobbin. The available length of winding space on the bobbin being 2.665 inches, the depth occupied will therefore be 1.925 inches. Minimum circumference of winding is 18.2 inches; maximum circumference is 30.096 inches; hence mean length of one turn is 24.15 inches = 2.01 feet. Resistance of one bobbin 9.22 ohms(at 60 C.) Watts lost in one bobbin 50. Weight of 2056 feet, in pounds 16 pounds. Bobbins being 2.67 inches long, with 30 inches external perif., the cooling surface is 152 square inches; watts per square inch .329; hence probable temperature rise 25 degrees C. N.B.--Total radiating surface of all the bobbins ought to be about 15 times the total kilowatts of output. Probable temperature rise 75 C. per watt per square inch.

CALCULATION OF SERIES WINDING

Assuming current density 1500 amperes per square inch of copper, the needful cross-section will be .04 square inches, and may be provided by strip copper of thickness .08 inch (=No..... B & S) and 0.5 inches broad. Mean length of one ture 23.72 inches; total length of strip required 355.8 inches = 29.6 feet for one pole, or 118.4 feet in total.



Scale-$\frac{1}{4}$"=1"
$\frac{3}{4}$" Tap
$\frac{1}{2}$" steel Bolt length=4"
$\frac{3}{8}$" machine screw
$2\frac{1}{2}$"
D=17.85"
$4\frac{3}{4}$"
$\frac{7}{16}$ Drilled Hole
6.22"
$1\frac{1}{8}$"
D-4
4"
R=$6\frac{3}{4}$"
$\frac{13}{16}$" Drilled Hole
$5\frac{5}{8}$"
$17\frac{1}{8}$"
$22\frac{3}{4}$"
$\frac{1}{2}$"
1.96"
$\frac{3}{4}$" Tap Depth=$1\frac{1}{2}$"
$6\frac{1}{4}$"
$22\frac{1}{4}$"
$\frac{3}{4}$"
$2\frac{3}{4}$"
1"
$1\frac{7}{16}$"
7"
$9\frac{7}{8}$"

Fig. 34

DIRECT CURRENT GENERATOR DESIGN

Fig. 35.

Scale - 1/2" = 1"

DESIGN OF ALTERNATING CURRENT GENERATOR.

In the design of alternating current generators there will be found many calculations and constants that are identical with the direct current generators, therefore when these constants apply, no further mention will be made pertaining to them.

FORM

The form of the commercial alternating current generator varies greatly with the speed as will be shown later. That is low speed generators are large in diameter and short in length, while for turbo-generators the reverse is true, figures 36 and 37 represent present day types of low and high speed generators.

RATING

Most alternators at the present time have their rating given in kilovolt amperes at a given voltage which at once settles the current which may safely be drawn from the armature, and for a three phase machine the current per terminal is

$K.V.A. = E\,I\,\sqrt{3}$ where E = the terminal pressure and I the current to be drawn per phase or

$K.W. = E\,I\,\sqrt{3}\,\cos\theta$ where E and I are the same quantities as given above and $\cos\theta$ is the power factor of the load. From either equation the line or terminal current may be found. In three phase generators the windings are usually connected in either delta or star.

Figure 36.

Figure 37.

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III WINDING

When Δ windings are used the following relations exist, see figure 38 . E = line voltage and is the voltage generated per phase winding, therefore the winding must be so made that the voltage generated per phase will be the terminal voltage of the generator.

$I' = \frac{I}{\sqrt{3}}$ where I' is the current in the windings of the generator and I is the current per terminal or line current. Hence the machine winding need only be made of sufficient cross section to carry the line current divided by the 3 $= \frac{I}{\sqrt{3}}$.

WINDINGS

When the windings are connected in Y , see figure 39 the line current will be equal to the current in the windings but the voltage generated per phase winding will be $\frac{E}{\sqrt{3}}$, and hence the following conditions exist.

Windings Connected	Line Voltage	Voltage per phase winding
Δ	E	E
Y	E	$\frac{E}{\sqrt{3}}$

Windings Connected	Line Current	Current in winding
Δ	I	$\frac{I}{\sqrt{3}}$
Y	I	I

NUMBER OF POLES

Since the frequency, f, = $\frac{poles}{2}$ x R. P. S. it is evident

that the number of poles is fixed by the speed at which the prime mover is to run. The tendency being to use very high speeds and consequently as few poles as possible, especially is this true with the turbo-alternators since the good efficiency of the steam turbine depends upon a very high rotative speed.

AIR GAP DIAMETER

So many alternators have been built and analyzed that the large companies rely to a certain extent on the data of previous machines but a very fair preliminary value may be taken from tables IV and V.

TABLE IV

	Engine Recip.		Water turbine		Steam turbine	
Cycles	25	60	25	60	25	60
Arm. diam. per pole	5	3	13	5	20	7.5
Arm. reaction about	3200	1800	8300	3200	13000	4800
No load ÷ arm. reaction	2.5	3	2.5	3	2.5	3
Regulation	8	6	8	6	8	6
% short circuit current	2.5	2.8	2.5	2.8	2.5	2.8

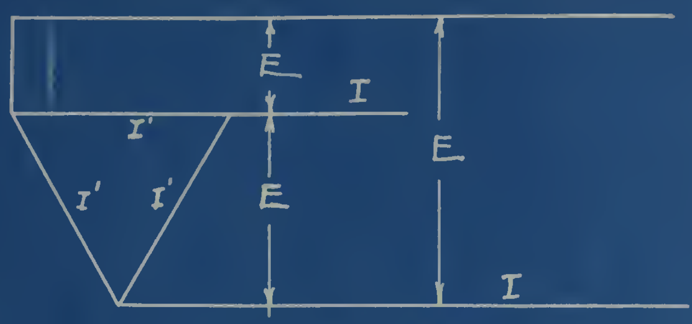


Fig. 38

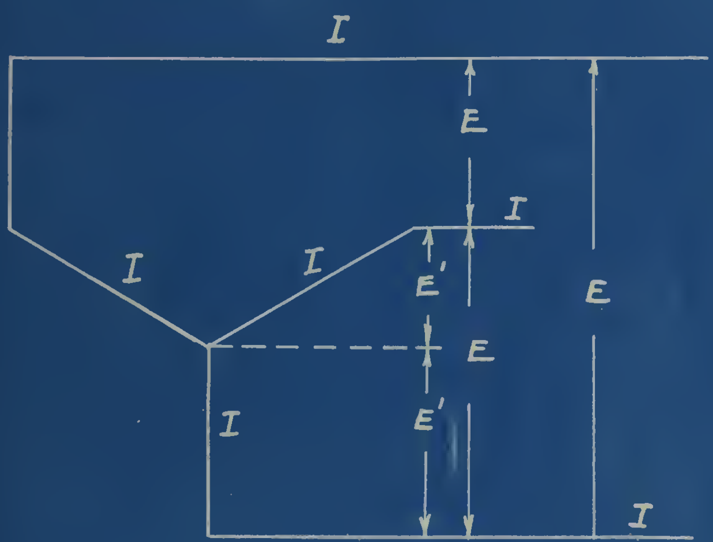


Fig 39

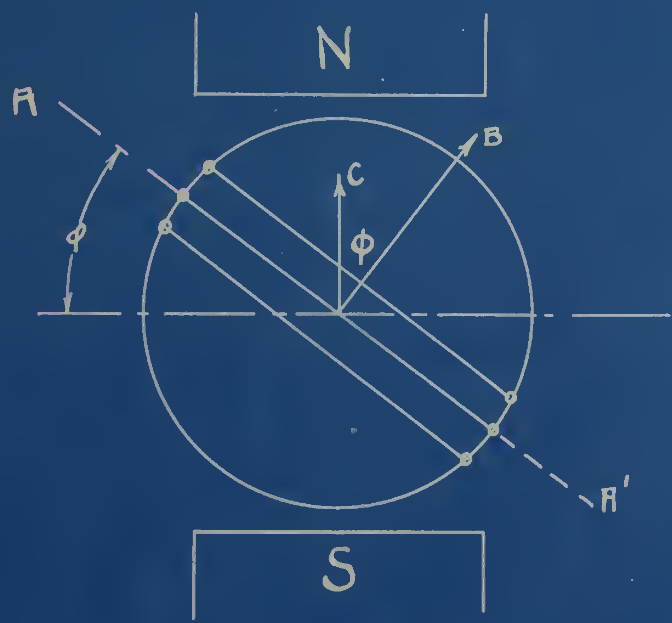


Fig. 40

TABLE V 25 CYCLES

Output in K.W.	Poles 4 Dia./pole	Poles 8 Dia./pole	Poles 12 Dia./pole	Poles 24 Dia./pole	Poles 32 Dia./pole
250	7.1	5.5	4.7	3.9	3.7
500	8.5	6.3	5.4	4.3	4.0
1000	9.8	7.15	6.3	5.0	4.6
2000	11.1	8.4	7.5	6.3	5.5
4000	12.9	11.2	10.0	8.6	7.7
6000	17.0	14.4	13.0	---	---
8000	20.0	17.0	16.0	---	---

60 CYCLES

250	5.2	4.5	3.5	3.0	2.9
500	6.0	5.2	4.7	3.6	3.2
1000	7.2	6.0	5.9	4.7	3.6
2000	8.5	7.0	6.3	5.3	4.8
4000	9.0	8.2	7.1	6.1	5.5
6000	9.0	9.0	8.0	6.9	6.4

It is seen from the above that table IV gives comparative values only while table V shows the variation of diameter per pole with kilowatts and also with number of poles which in reality tells whether the prime mover is to be steam engine, turbine, or water wheel.

POLAR PITCH

From the above table a trial value of diameter per pole can be obtained at once and hence since the polar pitch = T, where

$T = \pi \times$ Diameter per pole, it is seen that the pole pitch is also fixed.

ARMATURE REACTIONS

Let e = instantaneous value of e. m. f. and let $e = E \sin \varphi$. Let the current be lagging by angle α. Then $i = I \sin (\varphi - \alpha)$ (for inductive loading). Let N = number of turns in series per phase per pole = in figure 40, one half the turns represented by the winding.

In figure 40 let A - A' represent the position of the coil of one phase winding. Let vector B represent the direction of the total magneto motive force and vector C represent the direction of the demagnetizing magneto motive force. Again let N = the turns per pole per phase, in this case one half the total turns represented in the figure.

The total m. m. f. per pole of the coil will be equal to i N, in the position shown where i is the current in the coil at the position or time shown and the demagnetizing m. m. f. will be $i N \cos \varphi$, but $i = I \sin (\varphi - \alpha)$, hence the demagnetizing m.m.f.

$$= I N \cos \varphi \sin (\varphi - \alpha) \tag{84}$$

$$= I N \cos \varphi (\sin \varphi \cos \alpha - \cos \varphi \sin \alpha)$$

$$= I N (\cos \varphi \sin \varphi \cos \alpha - \cos^2 \varphi \sin \alpha)$$ = the demagnetizing m. m. f. at any instant provided the current lags the e. m. f. by angle α.

The average demagnetizing m. m. f. will be

$$\int_0^\pi \frac{\cos \alpha \sin \varphi \cos \varphi \, d\varphi}{\pi} - \int_0^\pi \frac{\cos 2 \varphi \sin \alpha \, d\varphi}{\pi} . \tag{85}$$

But since α is a constant angle the average demagnetizing m. m. f. =

$$I\ N \left[\frac{\cos \alpha}{\pi} \int_0^{\pi} \sin \varphi \cos \varphi \, d\varphi - \frac{\sin \alpha}{\pi} - \int_0^{\pi} \cos^2 \varphi \, d\varphi \right] \quad (86)$$

$$= I\ N \left[\frac{\cos}{\pi} \int_0^{\pi} 1/2 \sin 2 \varphi \, d\varphi - \frac{\sin \alpha}{\pi} \int_0^{\pi} 1/2 \, d\varphi + 1/2 \cos 2 \varphi \, d\varphi \right]$$

$= I\ N \dfrac{\sin \alpha}{2}$ and is the average demagnetizing m. m. f. per pole if of course I (effective) is used the equation becomes

$$= \frac{I(eff)\ N \sin \alpha}{2} \quad (87)$$

For a generator with n phases the instantaneous demagnetizing m. m. f. of the

First phase is $I\ n \cos \varphi \ (\sin \varphi - \alpha)$

Second phase is $I\ N \cos \left(\varphi + \dfrac{360^{\circ}}{n}\right) \sin \left(\varphi + \dfrac{360^{\circ}}{n} - \alpha\right)$

Third phase is $I\ N \cos \left(\varphi + \dfrac{2 \times 360^{\circ}}{n}\right) \sin \left(\varphi + \dfrac{2 \times 360^{\circ}}{n} - \alpha\right)$

where n = the number of phases

φ = the reference position of first phase

α = lag of the current behind the induced e. m. f.

The total demagnetizing action will be $\frac{n}{2} \sqrt{2}\ I\ N$ for full inductive load, (88)

where n = the number of phases

N = turns per pole per phase

I = the effective value of the current flowing in each winding.

TURNS PER PHASE.

Using the above equation and table number IV the trial value of turns to be used per phase may be determined at once, since from table number IV , the approximate armature reactions at full load for any type of machine may be found, thus for an n phase machine equate the value found in the table to the equation above or,

Arm. reactions from table = $\frac{n}{2}\sqrt{2}\,I\,N$, then since the effective value of the armature current per phase is known solve for N which will be the trial value of turns per pole per phase.

$N \times P$ = turns per phase and $n \times N \times P$ = total turns to be wound on the armature.

SLOTS.

Since the number of slots and hence the number of conductors per slots has a very great bearing on the ohmic reactance of the generator, this subject will be dealt with in detail under the heading armature reactance. In modern machines the number of slots per pole per phase vary from 2 to 12 or even more in some types of machines. The insulation of conductors of course varies with the voltage between conductors and the voltage to ground. Table gives approximate values of slot linings for different terminal voltages.

TABLE VI

Eff. volts per phase	Slot lining in m.m.
500	0.9
1000	1.4
2000	2.3
3000	2.9
4000	3.3
6000	4.0
8000	4.7
10,000	5.2
12,000	5.6

CURRENT DENSITY IN ARMATURE CONDUCTORS.

Since the thickness of insulation is a function of the voltage and since the temperature rise is dependent on the amount of insulation used and also the current density it is seen that the current density depends more or less on the voltage of the machine and while a current density as high as 2500 amperes per square inch may be used in moderate voltage generators, 1800 amperes per square inch in machines generating voltages greater than 10,000 should seldom be exceeded.

E. M. F. GENERATED.

It has been shown that when a coil of wire is rotated in a uniform magnetic field at constant velocity the average e.m.f. generated therein will be,

e (average) $= 4 \times \varphi \times N \times f \times 10^{-8}$ which is of course the formula for direct current machines where

φ = the total flux per pole

N = turns in the coil

f = the frequency

If the windings are so placed on the armature (as is usually the case) that the e. m. f. generated at every instant is proportional to the sine of the angle through which the coil has passed, it is evident that the terminal voltage will follow the sine law. It is easily proven that the average value of a sine curve is $\frac{2}{\pi}$ x its maximum value and the effective value of a sine curve is $\frac{1}{2}$ x its maximum value, therefore the ratio =

$$\frac{\text{effective sine}}{\text{average sine}} = \frac{\frac{E\ (max)}{2}}{\frac{2E\ (max)}{\pi}} = \frac{\pi}{2\ \ 2} = 1.11. \tag{89}$$

Hence for effective e. m. f. ,

E (eff) $= 4 \times 1.11 \times \varphi \times N \times f \times 10^{-8}$ and may be applied to the alternator by using,

N = turns per phase

φ = flux per pole

E = efficiency e. m. f. per phase.

This formula only holds when all the turns of the winding are simultaneously linked with the total flux per pole. This is usually the case in a three phase alternator see figure 41 , but in a two phase machine the winding of each phase occupies one half the polar pitch and it may be seen in figure 42 that the innermost turns are not quite interlinked with the total flux.

This fact is expressed by stating that in a three phase winding the spread of the winding is 33.3% of the polar pitch and that in a two phase winding it is 50% of the polar pitch. The correction is so small for this condition that no further mention of it will be made.

Another condition which enters due to the spread of the coils is that caused by the electro motive force generated in each individual coil not being in phase with the e. m. f. generated in any other coil and hence the total e. m. f. of the coil will be the vector sum and not the arithmetical sum.

Consider the six coils 1, 2, 3, 4, 5 and 6 and let their e. m. f.'s be represented by ab, bc, cd etc. If there were an infinite number of coils the sum of their e. m. f. would be represented by the half circle and the vector sum by the diameter, (see figure 46) and the ratio of the terminal e. m. f. as represented by a - g to the total generated e. m. f. would be,

$$\frac{\frac{1}{2}\text{ circumference}}{\text{diameter}} = \frac{\frac{\pi D}{2}}{D} = \frac{\pi}{2}\,. \tag{90}$$

Next consider a belt of conductors covering 60 degrees as is the case in a three phase alternator figure 47 . In this case the line a - c represents the coil e. m. f. and the arc a - b - c represents the total induced e. m. f. The ratio of terminal e. m. f. to total will be in the ratio of the cord to the arc,

$$\text{Cord} = 2\ R \sin\frac{\alpha}{2}$$

$$\text{Arc} = \frac{2\pi R}{6}$$

$$\text{Ratio} = \frac{2\ R \sin\frac{\alpha}{2}}{\frac{2\ \pi\ R}{6}} = \frac{6 \sin\frac{\alpha}{2}}{\pi}$$

For $\alpha = 60$

$$\text{Ratio} = \frac{3}{\pi}$$

For $\alpha = 90$

$$\text{Ratio} = \frac{6}{2\ \pi}.$$

The e. m. f. formula must therefore be corrected for this loss.

Let K = ratio of cord to arc for any case then $E = 4.44\ K\ f\ \varphi\ N\ 10^{-8}$.

In the above statements a full pitch winding was considered, as represented in figure 43 where the throw of the coil is equal to the polar pitch. It sometimes is desirable to have a lesser pitch as shown in figure 44 . For such a winding there must be employed still another constant K' which is called the winding pitch factor if the winding pitch is x% of the full pitch, then

$$K' = \sin\left(\frac{x}{100} \times 90\right) \text{ as will be shown below.} \qquad (91)$$

When the coil is in the position represented by A - A', figure 48 we have a full pitch winding and all the flux is interlinked by the coil. Now let the coil be wound as shown by B - B, then β = 90 and the coil is said to have half pitch.

From the figure, $\alpha = (90^\circ - \frac{\beta}{2})$

Let φ = the total flux per pole and let

φ_0 = the fluc enclosed by the coil for any spacing.

Then $\varphi_0 = \varphi \cos$

$$= \varphi \cos\left(90 - \frac{\beta}{2}\right) \qquad (92)$$

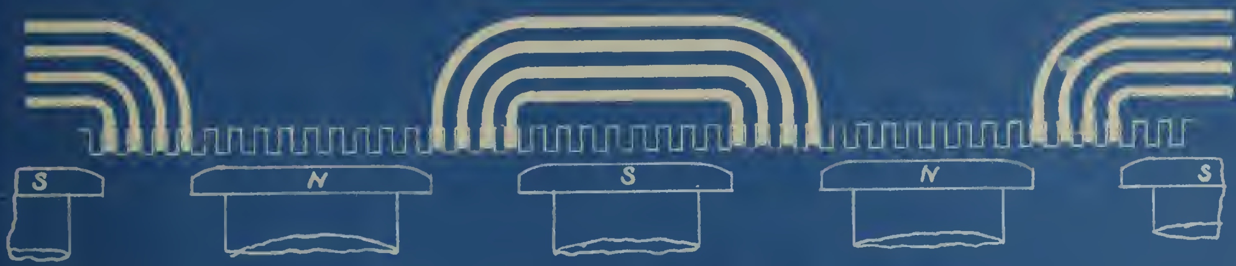


Fig. 41

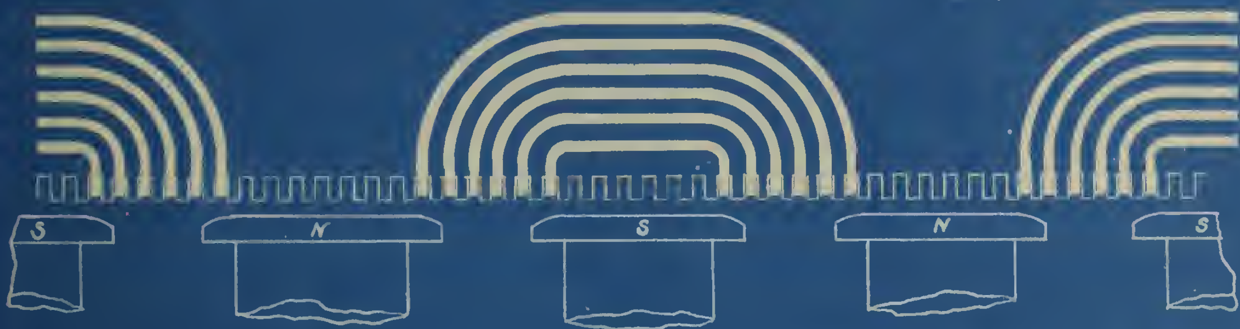


Fig. 42


Fig 43

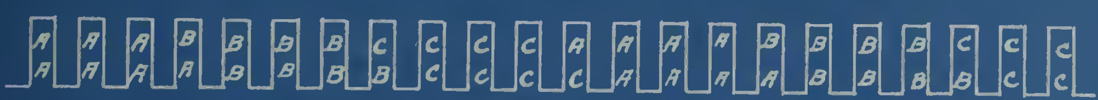


Fig. 44


Fig. 45

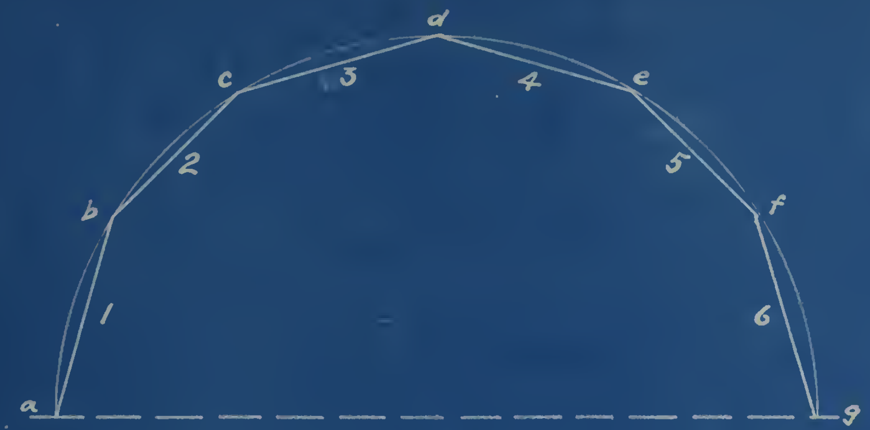


Fig. 46

Fig. 47

Fig. 48

$$= \varphi\,(\cos 90 - \cos\frac{\beta}{2} + \sin 90 \quad \sin\frac{\beta}{2})$$
$$= \varphi \sin\frac{\beta}{2}.$$

Therefore the effectiveness of the total flux has decreased by the amount

$$\sin\frac{\beta}{2} = K' = \sin\left(\frac{x}{100}\,90^\circ\right) \qquad (93)$$

and the true e. m. f. formula becomes,

$$E = 4.44 \times f \times \varphi \times N \times K \times K' \times 10^{-8}. \qquad (94)$$

Since fractional pitch is very seldom used the factor K' does not often enter especially is this true in multipolar machines and the e. m. f. is usually based on the formula,

$$E = 4.44 \times f \times \varphi \times N \times 10^{-8}. \qquad (95)$$

This formula will give the no load value of flux per pole and at full load an additional flux must be produced to compensate for the I R drop in the armature. Reactance and armature reactions being omitted for the present. The value of flux thus calculated will be the flux which must be cut by the armature conductors and hence to obtain this flux the flux carried by the field core and yoke must be increased by an amount equal to the leakage. The value of leakage being dependent on the same factors as for the direct current machine see page 14 . Where round rotar machines are used these values of leakage coefficients may be materially reduced.

MEAN LENGTH OF ARMATURE TURN.

The formula for direct current machines was given as, mean length of armature turn = $2\,L + 10\,\frac{D}{P}$, but for alternators with their great range of voltages this formula does not always apply

due to the increased length of end turns with high voltages. The following may more safely be used,

$$\text{Mean length of turn} = 2\ L + K\ T', \tag{96}$$

Where L = total length of armature iron

T'= pole arc

K = a constant depending on the terminal voltage as given in table

TABLE VII

Terminal Volts	K.
500	2.5
1000	3.0
2000	3.5
4000	4.0
6000	4.5
8000	5.0
10000	5.5
12000	6.0

All of the equations have now been developed for determining the flux per pole, and hence the proceedure is exactly as in the direct current generator, to determine the length of machine and cross section of all parts, as yoke, pole core, armature body etc. using approximately the same flux densities as have already been given.

AIR GAP.

The area of the air gap may be found as in the case of the direct current generator.

The length of the gap is found by the conditions given in table IV . That is the steady value of the short circuit current should be limited to 2 1/2 times the full load current. This may be done by making the ampere turns of the field necessary to force the flux across the air gap at no load equal approximately to 2 1/2 times the armature reactions at full load armature current.

Hence since the arpere turns for the air gap = $.313\,\beta\,L_g$, make L_g (the length of the air gap) such that it will require 2 1/2 times the armature reactions at full load.

FIELD CORE LENGTH.

The length of the field core may next be settled (assuming definite pole) giving due attention to the heating, as the watts per square inch radiating surface must be kept as low as 5 to 1, except in cases where the peripheral speed is very high or the coils are wound in very thin layers. If the generator is to be built with a stationary field the field current density should not exceed 1000 amperes per square inch while with a revolving field this value may reach as high as 1500 amperes per square inch in well ventilated generators where the peripheral speed is not so high that the windings must be completely inclosed to hold them in place. Again, since with inductive load there is needed a greater field current to produce normal terminal voltage, it is well to start with a no load field current density equal to about 35% of the maximum allowable density, this will then make ample allowance for the increased field current with full load at a power factor considerably less than unity. The exciting voltages for alternating current generators may be taken as 110 volts up to 1500 K. W. and 220

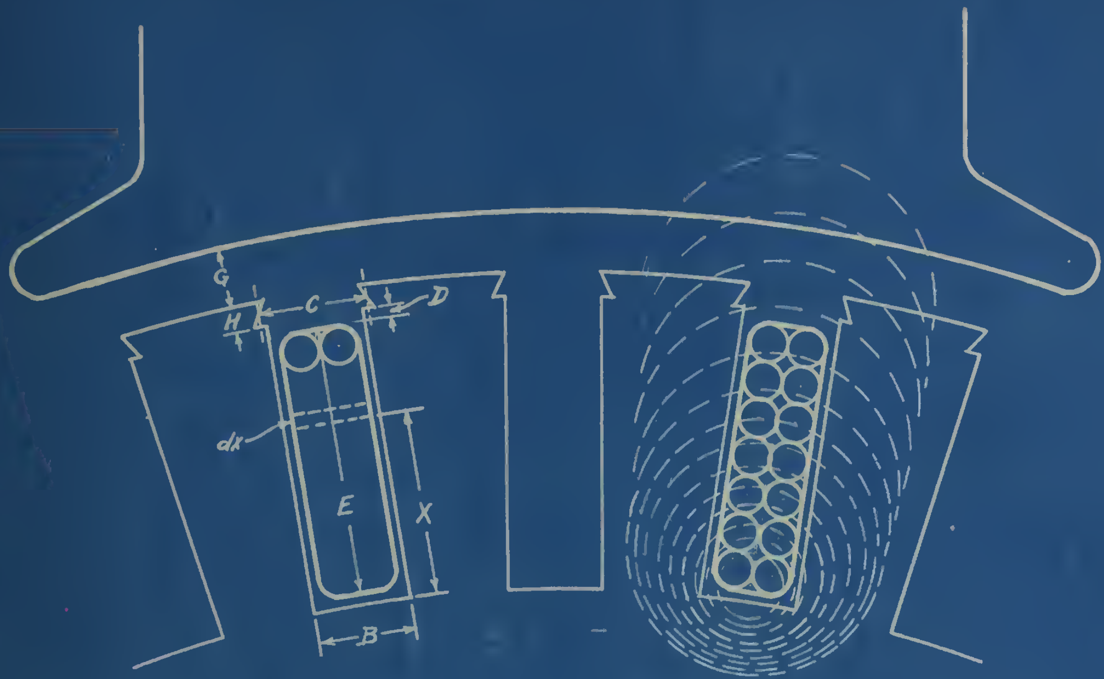


Fig. 49

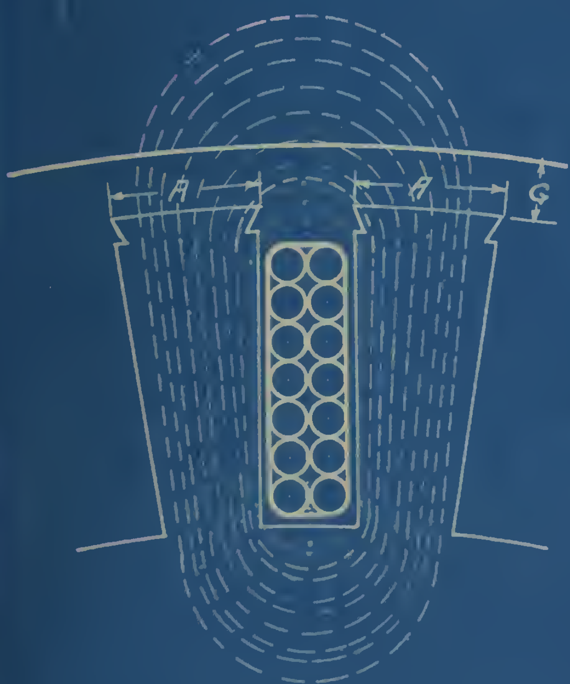


Fig. 50

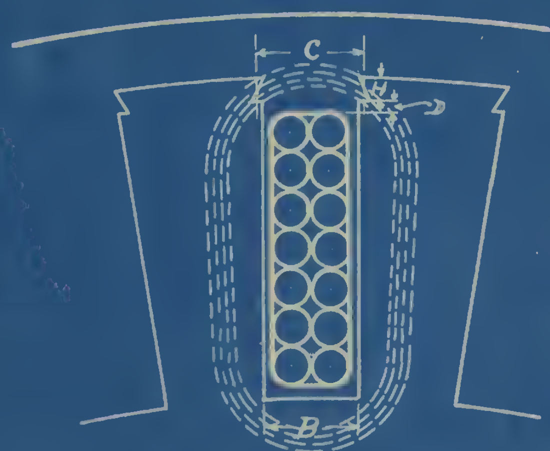


Fig. 51

volts for machines of larger capacity.

Having settled on the densities to be used in each part of the magnetic circuit and also having determined the length of each section the open circuit saturation curve may now be calculated exactly as in the case of the direct current machine and this curve should be carefully plotted.

ARMATURE REACTANCE.

Using figure 49 let us consider the self inductance of the armature conductors which lie in a slot under the pole face. It is evident from the figure that there will be a flux set up which will surround the conductors in each slot and which will be caused by the current flowing in these conductors. Then since $\text{flux} = \frac{\text{m. m. f.}}{\text{Reluctance}}$ we may write

$$\varphi = \frac{\text{m. m. f.}}{R} = \frac{.4\pi \, N \, I}{\frac{L}{\mu \, a}} \tag{97}$$

since m. m. f. = $.4\pi \, N \, I$ and reluctance = $\frac{\text{length of path}}{\mu \text{ times the area}}$

Now let us call this total flux φ_0 and since the length of magnetic path for this flux is not constant the flux will be divided into five paths and designated as φ_1, φ_2, φ_3, φ_4, φ_5. Let φ_1 = flux as represented in figure 50 . The magneto motive force producing this flux is therefore equal to $.4\pi \, N \, I \sqrt{2}$, where I is the effective value of current in the conductor or conductors of the slot. The reluctance of the air gap is so great in comparison to the reluctance of the iron path through which this flux passes that only the length of the air circuit need be considered.

Therefore the reluctance $= \frac{L}{a}$ and per unit length of armature a will be represented by A and the length of path by 2 G, hence

$$\varphi_1 = \frac{.4\pi\ N\ I'\ \sqrt{2}\ A}{2\ G}$$ where A and G are in centimeters if A and G are expressed in inches,

$$\varphi_1 = \frac{3.2\ \sqrt{2}\ I\ N\ A}{2\ G}$$

Now let φ_2 = flux through the section H, figure 51 .

Then,
$$\varphi_2 = \frac{3.2\ \sqrt{2}\ I\ N\ H}{C}$$

Let φ_3 = flux through the section D

$$\varphi_3 = \frac{3.2\ \sqrt{2}\ I\ N\ D}{B}$$

Now let φ_4 be the flux which cuts across the copper conductors in the slot figure 52 . From the figure it is seen that the conductors which lie in the bottom of the slot will be interlinked with all of the flux φ_4 but the top conductors will only be interlinked with a portion of this flux and again the m. m. f. of the bottom conductors will tend to produce a flux surrounding the bottom conductors only.

Now let us investigate the flux $d\varphi_4$ through the section dx.

$$d\varphi_4 = \frac{3.2\ \sqrt{2}\ I\ N\ \frac{x}{E}\ dx}{B}$$ and since this flux only cuts or interlinks $\frac{x}{E}$ N conductors, an equivalent flux cutting all the conductors would be

$$d\varphi_4 = \frac{3.2\ \sqrt{2}\ I\ N\ \frac{x^2}{E^2}dx}{B} \tag{98}$$

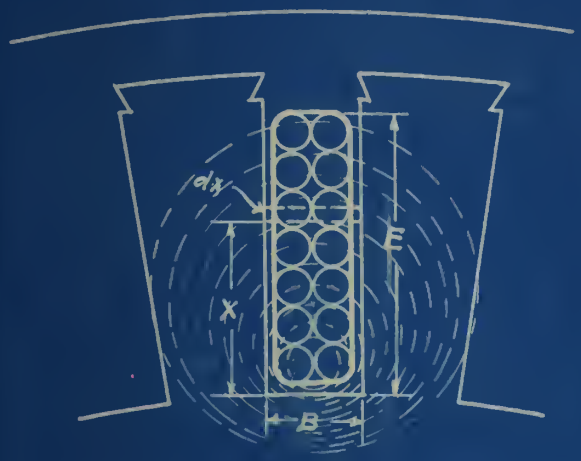


Fig 52

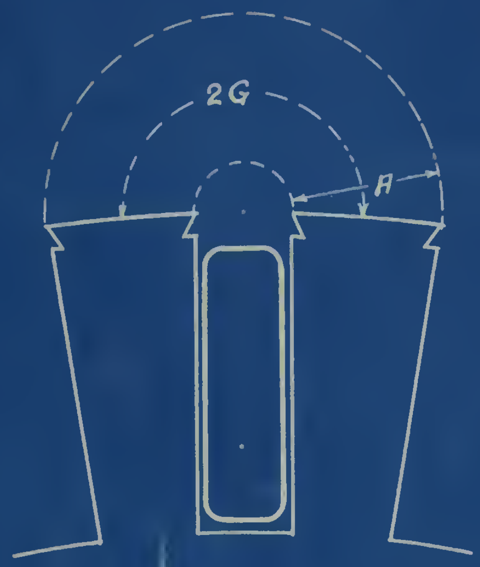


Fig. 53

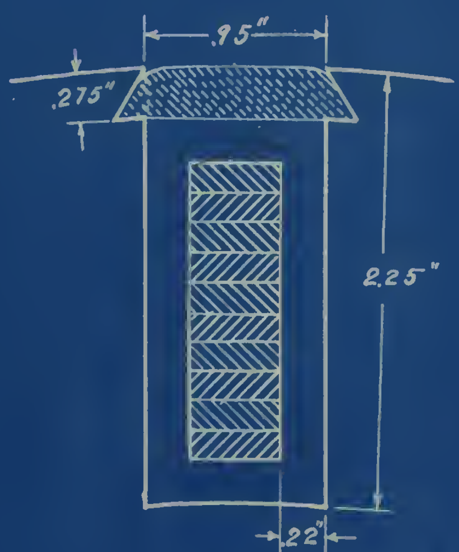


Fig. 54

$$\varphi_4 = \int d\varphi_4 = \int_{x=0}^{x=E} \frac{3.2\sqrt{2}\, I\, N\, x^2\, dx}{B\, E^2} \tag{99}$$

$$\varphi_4 = \frac{3.2\sqrt{2}\, I\, N\, E}{3\, B} \tag{100}$$

Now all the flux has been found except that due to the end turns, φ_5, which is very difficult to calculate and has been found to be about 1/10 as great per unit length of end turn as the flux produced in the iron part of the circuit or,

$$\varphi_5 \text{ per inch} = 1/10\ (\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4)$$

It must be remembered that φ_1, φ_2, φ_3 and φ_4 have been deduced per unit length of armature and,

$$\varphi_0 = 3.2\sqrt{2}\, I\, N\, \left(\frac{A}{2\, G} + \frac{H}{C} + \frac{D}{B} + \frac{E}{3\, B}\right) + \varphi_5$$

where N = conductors in one slot,

I = effective current flowing,

A, B, C etc. are as shown in figure 49 all dimensions being in inches.

Now let L = the effective length of the armature. Then the flux produced by the conductors in the iron part of the circuit will be

$$3.2\sqrt{2}\, I\, N\, \left(\frac{A}{2\, G} + \frac{H}{C} + \frac{D}{B} + \frac{E}{3\, B}\right) L$$ and if we let L_t =

length of one end turn,

$$\varphi_5 = .32\sqrt{2}\, I\, N\, \left(\frac{A}{2\, G} + \frac{H}{C} + \frac{D}{B} + \frac{E}{3\, B}\right) L_t$$

and φ_0 will be,

$$\sqrt{2}\, I\, N\, \left(\frac{A}{2\, G} + \frac{H}{C} + \frac{D}{B} + \frac{E}{3\, B}\right)\ (3.2\, L + .32\, L_t).$$

Now if there are S slots per pole per phase the total φ_0 will be per phase

$$\varphi_0 = \sqrt{2}\, I\, N\, S \left(\frac{A}{2\,G} + \frac{H}{C} + \frac{D}{B} + \frac{E}{3\,B}\right)(3.2\, L + .32\, L_t). \quad (101)$$

Since with more than one slot per pole per phase the flux produced by the conductors in another slot thereby increasing the inductance, φ_0 must be multiplied again by a constant K, as found from table VIII.

TABLE VIII

Slots per pole per phase =	1	2	3	4	5	6	10
K =	1	1.3	1.4	1.5	1.6	1.7	2

$$\varphi = \sqrt{2}\, I\, N\, S\, K \left(\frac{A}{2\,G} + \frac{H}{C} + \frac{D}{B} + \frac{E}{3\,B}\right)(3.2\, L' + .32\, L_t)$$

But since $L_1 = \dfrac{N\,\varphi_0}{\sqrt{2}\, I\, 10^8}$ = Inductance per phase

We have $L_1 = \dfrac{N^2 S\, K}{10^8}\left(\dfrac{A}{2\,G} + \dfrac{H}{C} + \dfrac{D}{B} + \dfrac{E}{3\,B}\right)(3.2\, L + .32\, L_t)$

$$\text{And } x = 2\pi f L_1 = \frac{2\pi\, f\, N^2\, S\, K}{10^8}\left(\frac{A}{2\,G} + \frac{H}{C} + \frac{D}{B} + \frac{E}{3\,B}\right)(3.2\, L + .32\, L_t) \quad (102)$$

Where x = the reactance of one phase in ohms.

The above expression then gives a method by which the reactance of a machine may be found in terms of its dimensions. This expression of course applies when the slot is directly beneath the pole shoe in definite pole machines and for any position for machines with a round rotor. The value of x must be found for the phase windings of definite pole machines with the slot placed between the poles. Practically the only factor that will change is the term represented by $\frac{A}{2\,G}$. In this case A may be assumed to remain constant but 2 G must be taken as shown in figure 53.

The value of reactance calculated with 2 G taken as in figure 53 is denoted by x'.

SLOTS PER POLE PER PHASE.

By inspection of equation 102 it can be seen that the reactance for a given machine varies as the square of the conductors in a slot and only directly as the number of slots, also the shape of the slot has an important bearing on the reactance, hence by properly proportioning the slot and air gap any reasonable reactance may be obtained.

SHAPE OF SLOT.

Figure 54 represents a typical slot for a large alternator. The slot contains 10 conductors and has a slot insulation corresponding to a terminal voltage of 12,000 volts. The space factor of this slot is about .29.

ARMATURE WINDINGS.

Fractional pitch windings were discussed on page 78 and figures 43, 44 and 45 show three windings with 100%, 91.5% and 83.5% pitch respectively. An examination of figure 43 will show that the conductors in any one slot belong exclusively to one of the three phases A, B and C, while in the case of fractional pitch windings there are a certain number of slots containing conductors belonging to two phases and as a consequence the winding should be of the two layer type. This leads to the use of lap windings such as are used in direct current machines. But full

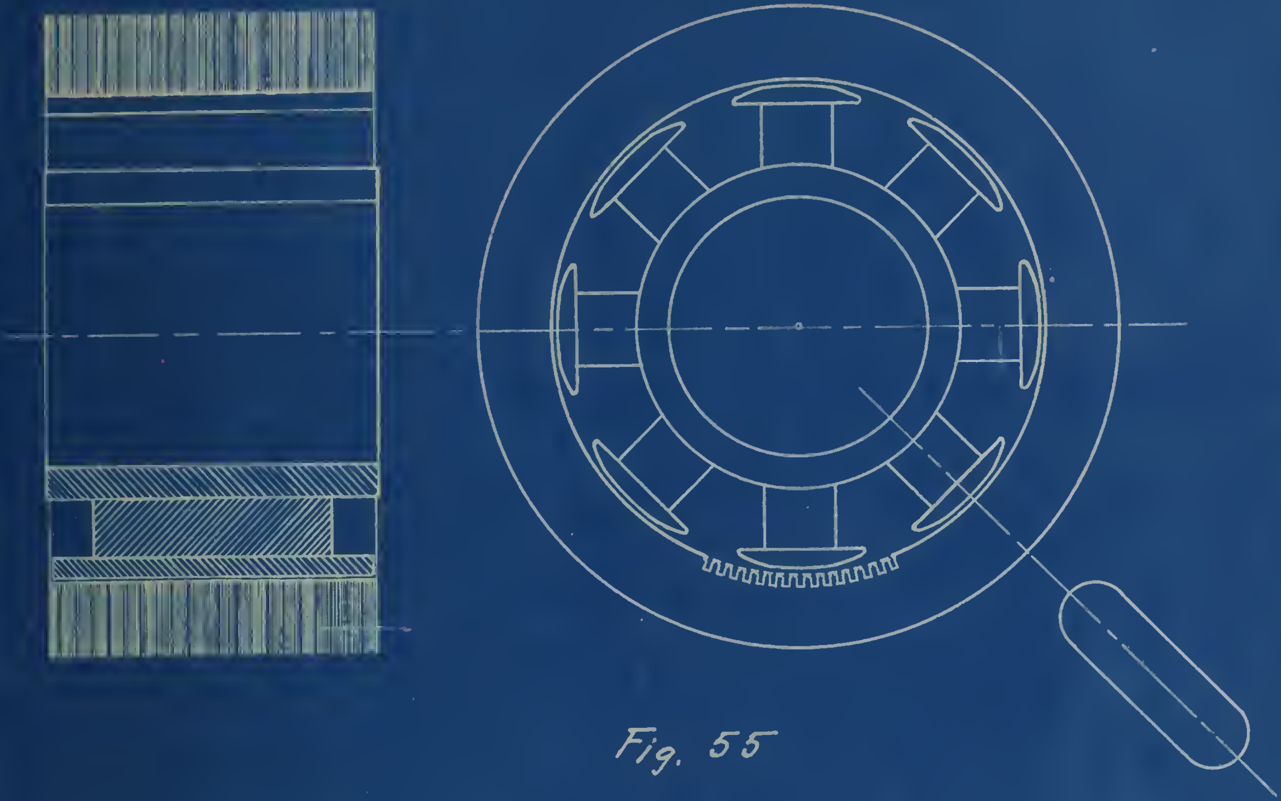


Fig. 55

Fig. 56

Fig. 57

pitch windings may be carried out either as lap or spiral windings and consequently as single or two layer windings, in figures 58 and 59 .

Again windings may be carried out in what is called whole-coiled in which, for any one phase, there is an armature coil opposite each field pole, as shown in figure 57 , or they may be carried out as half-coil winding, there being, for any one phase only one armature coil per pair of poles instead of one armature coil per pole. A half coil winding is shown in figure 56.

It is usual to employ half coil windings for three phase windings where the spiral type is adopted, but lap windings are inherently whole coil windings, as may be seen from figure 58.

It is mechanically desirable to employ fractional pitch windings in bipolar designs, as otherwise the arrangements of the end connections presents difficulties. In most instances of designs for more than two poles, a full pitch winding is satisfactory.

Figure 55 represents the general shape of revolving pole alternator (definite pole type) and shows the mean length of magnetic path per pole.

CALCULATION OF FIELD AMPERE TURNS
FULL LOAD AND VARIOUS POWER FACTORS.

Let x = reactance of one phase as measured with windings directly beneath the pole shoe.

x_1 = reactance of one phase as measured with windings between pole shoes.

r = resistance per phase.

Fig. 58

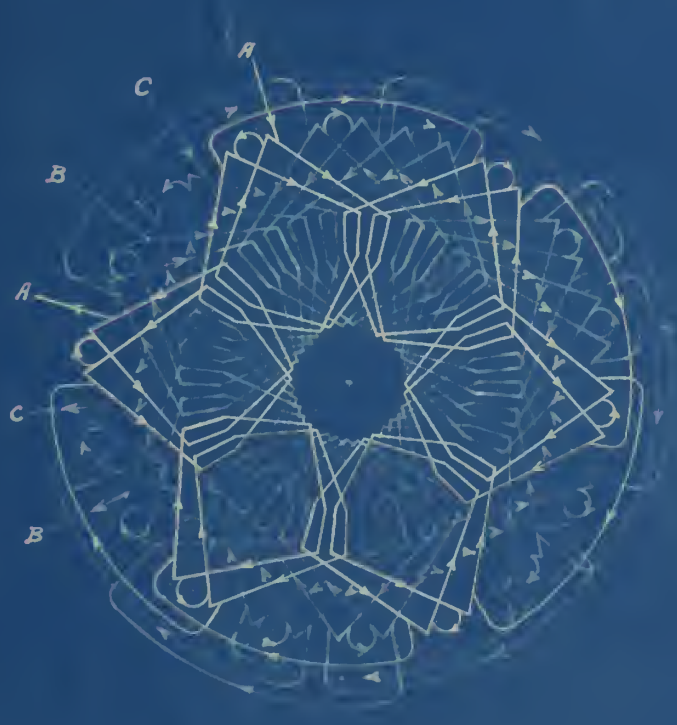


Fig 59

e = terminal voltage.

E_0= Induced voltage.

The armature reactions for three phase machines may be expressed as $1.5\sqrt{2}\ I\ N$, or for an n phase machine $= \frac{n}{2}\sqrt{2}\ I\ N$. This, of course assumes a concentrated winding.

The diagram for the alternator is shown in figure 61 which shows the various vectors assuming a lagging current. Since lagging current is expressed as,

$\dot{I} = i + j\, i_1$ and impedance

$\dot{Z} = r + j\, x'$ where x is inductive reactance.

The total induced e. m. f. $= e + \dot{I}\,\dot{Z}$,

$= e + (i + j\, i_1)(r + j\, x)$

$= e + i\, r - i_1 x + j\,(i\, x + i_1 r)$ (103)

where $e + i\, r - i_1 x$ = real component of the voltage and $(i\, x + i_1 r)$ = out of phase component.

Now let us turn to the open circuit saturation curve which shows the value of field ampere turns plotted against terminal volts.

When a load is placed on the machine the value of field excitation must be changed on account of the armature resistance, reaction and self induction.

The armature reaction may demagnetize, magnetize or shift the field flux from one side of the pole to the other or may combine both effects.

The energy current is a maximum when the e. m. f. is a maximum and therefore reaches this value when the slot is directly beneath the pole and hence has only a shifting tendency.

If the magnetic reluctance is constant all around the

armature then the shift of the field flux may be found directly from a vector diagram. This would be the case in a round rotor alternator but when definite poles are used the magnetic reluctance is not constant and the shift has been found to be from .5 to .66 the value as obtained for a round rotor machine or as obtained for a vector diagram assuming equal reluctance.

The wattless component of current, however reaches its maximum when the slot is midway between the poles and since the armature reaction due to this component of current is either magnetizing of demagnetizing depending on whether the current is leading or lagging and therefore is the same for both types of machines.

Now let $M \dot{I}$ = the total armature reaction then since $\dot{I} = i + j i_1$ (for lagging current) the armature reaction $= M i - j M i_1$ for a round rotor alternator.

For a deninite pole machine armature reactions will be, $M i + j M_1 i_1$, where $M = .5 M_1$ to $.66 M_1$, these components are represented in figure 61.

The self induction of the alternator is caused by flux which is set up by the armature current and which does not interlink with the main flux. In a definite pole machine this changes with the position of the armature winding in regard to the pole when the power component of current is a maximum the self induction is a maximum and when the wattless component of current is a maximum the self induction is a minimum.

A method for calculating the self induction for various positions of the slot has been given. In the diagram two other factors are represented i.e. n and n_1, which represent two values of m. m. f. which show the phase position of the true m. m. f. of

the field which is in phase with the field flux. n and n_1 are therefore expressed in ampere turns. n is found directly from the open circuit saturation curve and is that value of ampere turns which corresponds to the real voltage $(e + i\,r - i_1 x)$, n_1 is found by the proportion,

$$\frac{(i\,x + i_1\,r)}{(e + i\,r - i\ \ x)}\ n.$$

Now since the flux is 90 ahead of the induced e. m. f. we can represent this flux in complex quantities by multiplying by $+J$, or since $F =$ the vector sum of $(+n_1 + j\,n)$, and $-M\,i - j\,M_1\,i_1$, which represents the armature m. m. f. or,

$$F = -M\,i' - n_1 + J(n - M_1\,i_1)$$

$$F^2 = (-n_1 - M\,i)^2 + (n - M_1\,i_1)^2$$

$$F = \sqrt{(n_1 + M\,i)^2 + (n - M_1\,i_1\,)^2} \qquad (104)$$

This gives a method of calculating the total field ampere turns for any power factor and any armature current.

EFFICIENCY

The efficiency can now be calculated by the following method.

Assume non-inductive load and calculate,

1. I^2R loss of armature at any load,

2. I^2R of field at any load,

3. Hysteresis loss may be found by calculating the weight of the armature stampings and taking the loss per pound for the

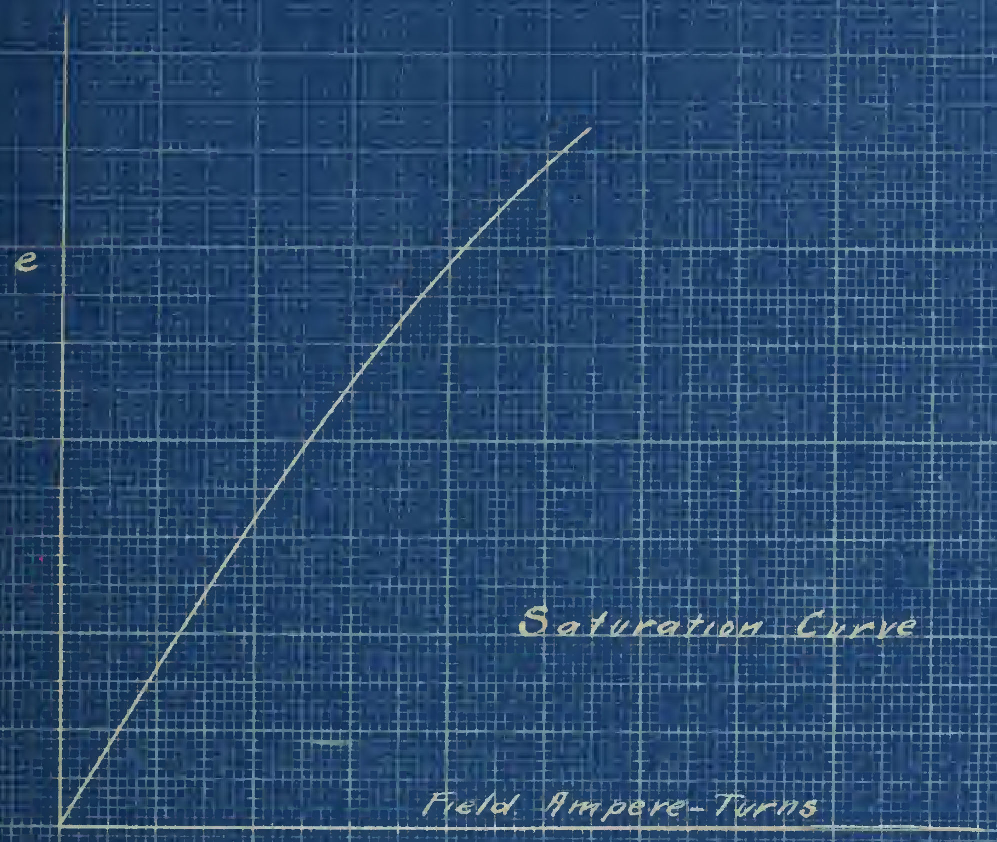
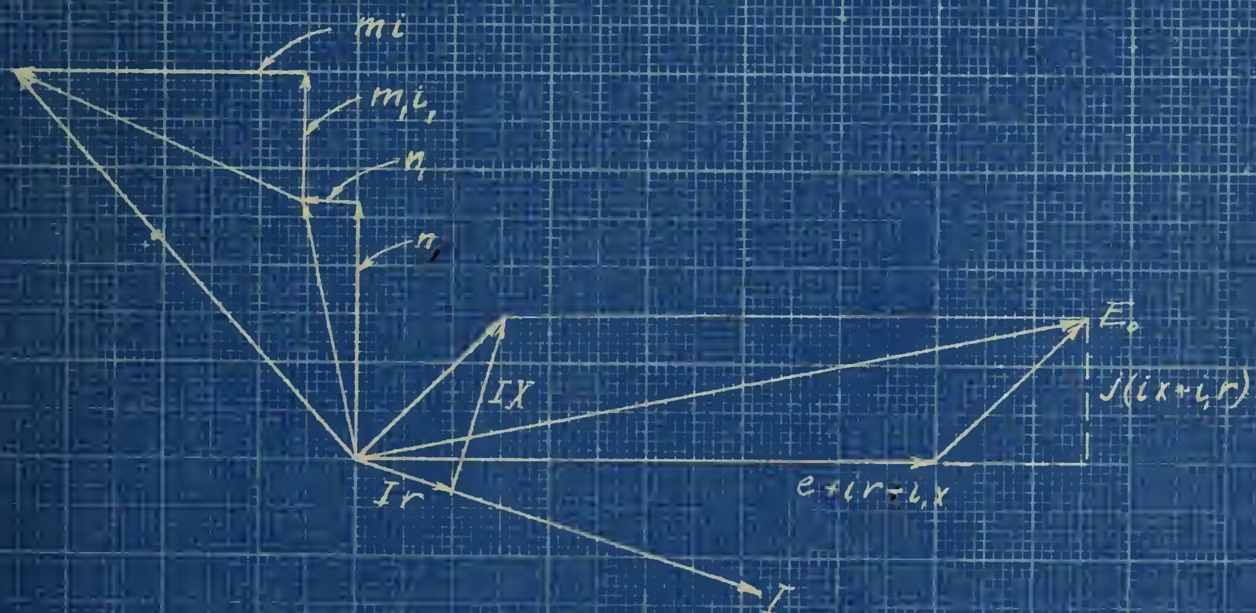


Fig 60



Vector diagram of A.C. Generator
Fig. 61

curve given in the direct current design.

4. Windage and friction must be assumed.

$$\text{Eff} = \frac{\text{Output}}{\text{Output} + \text{losses}} \quad .$$

Although in most cases the friction and windage losses for direct connected generators are charged to the prime mover, it will not be done so with the machine designed.

CALCULATION OF A TURBO-ALTERNATOR.

Three phase - 15-- K. W. - 60 cycles - 6 poles - 2300 volts - Y connected - 1200 R. P. M.

Diameter = K x N_o of poles.

From Table K = 7.5 D = 7.5 x 6 = 45" diameter.

Peripheral speed = $\dfrac{\pi\ d \times r.\ p.\ m.}{12}$ = 14,000' per minute.

P = $\sqrt{3}$ E I, where E = line voltage, I = line current.

$I = \dfrac{P}{\sqrt{3}\ E} = \dfrac{1500000}{3 \times 2300}$ = 377 amp.

Armature reactions for turbo-alternator between 3750 - 4500.

Take 4000 ampere turns.

Arm Reaction = 1.5 2 I N $N = \dfrac{4000}{2\ 1.5 \times 377}$ = 5 turns per pole per phase.

No load ampere turns 2.5 x 4000 = 10,000.

Since three phase - 6 poles, total turns on armature = 5 x 3 x 6 = 90 turns = 180 conductors.

Take two conductors per slot makes 90 slots.

Pitch of slot and tooth = $\dfrac{\pi \times 45}{90}$ = 1.57".

From table insulation must be .14" thick.

Assuming 550 mils per ampere, for 377 amperes there must be 207,000 mils = .1625 square inches.

Take width of slot .57" tooth 1".

Width of conductor = .57 - (2 x .14) = .29".

Height of copper must be $\dfrac{.1625}{.29}$ = .56".

Dimensions of conductor .29" x .56".

Depth of tooth = (2 x .56) + (3 x .14) + .32 = 1.86" call 1 7/8"

Width of tooth at top 1"

Width of tooth at base 1.04"

Width of tooth at notch .8"

Average width .95"

Use 4 air ducts 3/8" total = 1.5"

TOTAL LENGTH OF ARMATURE

Flux = $\frac{E \times 10^8}{4.44 \text{ f N}}$. E =er coil = $\frac{2300}{3}$ = 1330 volts.

$$\phi = \frac{1330 \times 10^8}{30 \times 4.44 \times 60} = 16.6 \text{ megalines.}$$

Assume density in teeth = 100000 lines per square inch.

Assume pole arc = 65%.

Length of armature covered by one pole = $\frac{.65 \times \pi \times 45}{6}$ = 15.3".

Number of teeth covered by one pole = $\frac{15.3}{1.57}$ = 10.

Area of teeth under one pole = $\frac{16600000}{100000}$ = 16.6 square inches.

Armature length = $\frac{166}{10 \times .95}$ = 17.5". Allow .1 for insulation.

Air ducts = 1.5".

Total length of armature = $\frac{17.5}{.9}$ + 1.5 = 21".

Dimensions of pole shoe = 15.3" x 21" = 321 square inches.

Approximate area of air gap = $\frac{321 + 166}{2}$ = 234 square inches.

Area of air gap = Eff. area of pole shoe + area of teeth at surface divided by 2.

Eff. area of pole shoe = (reg. arc + K x length of air gap) length of armature K from table = 2.2.

Eff. area of pole shoe = (15.3 + 2.2 x 7/16) 21 = 342 square inches.

Eff. area of air gap = $\frac{342 + 166}{2}$ = 254 square inches.

LENGTH OF AIR GAP = L_g

A T = 2 1/2 x 1.5 $\sqrt{2}$ x N I = 2 1/2 x 4000 = 10000.

A T = .313 B L_g $B = \frac{\varphi}{\text{Eff. A}} = \frac{16600000}{254}$ = 65400 lines per sq. in.

$L_g = \frac{10000}{.313 \times 65400}$ = .489 inches call .5 inches

CALCULATION OF REACTANCE

$$L_1 = N^2 S K \left(\frac{A}{2 G} + \frac{H}{B} + \frac{D}{C} + \frac{E}{3 B}\right) (3.2 L + .32 L_t) 10^{-8}$$

$$L_1 = 4 \times 15 \times 1.6 \left(\frac{1}{2 \times .489} + \frac{.14}{.57} + \frac{.8}{.07} + \frac{1.875}{.3 \times 1.04}\right) (3.2 \times 21 + .32 \times 29)$$

See drawing of slot for dimensions

L = length of arm conductor (see next calculations)

$$L_1 = \frac{96 (1.025 + 1.193 + .6) (67.2 + 9.28)}{10^8}$$

$$L_1 = \frac{2.055}{10^8} = .000205 \text{ henries}$$

x = 2 π f L = 377 x .000205 = .0774

$$\frac{.0774 \times 37.7}{1330} = .0219 = 2.19\%$$

x' = reactance for full lagging current.

Since there is no pole shoe above teeth, length of 2 G in this case = .785 x π = 2.465 inches.

$$L = \frac{96 \left(\frac{1}{2.465} + 1.193 + .6\right) 76}{10^8}$$

$$= \frac{96 \times 2.19 \times 76}{10^8} = .00016 \text{ henries}$$

$$x' = \frac{377 \times .00016}{1330} = .0604$$

$$x' = \frac{377 \times .0604}{1330} = .0172 = 1.72\%$$

ARMATURE RESISTANCE

Length of arm coil = 2 L + k T'

L = length of armature T' = pole arc. k from tables = 4.

Length = 2 x 21 + 4 x 15.3 = 103.2 inches

Resistance per foot = .00005

Resistance of armature = 2 x 30 x 103.2 x .00005 = .01294

Call .013 per phase.

$$\frac{.013 \times 377}{1330} = 3.69\ \%$$

CALCULATION OF DIMENSIONS AND LENGTH OF PATHS

Radial depth of armature assuming 80000 lines per square inch =

$\frac{\phi}{\beta L}$. $\phi = \frac{16.8}{2} = 8.4$ megalines since flux divides.

L = 21 inches.

$$\text{Radial depth} = \frac{8400000}{80000 \times 21} = 5 \text{ inches.}$$

Entire depth of arm body = 5 + 1 7/8 = 6 7/8 inches.

$$\text{Mean diameter} = \frac{45 + 3\ 3/4 - 10 + 45}{2} = 51.87 \text{ inches.}$$

$$\text{Mean mag. length of path in arm body} = \frac{51.87 \times \pi}{2 \times 6} = 13.57 \text{ inches.}$$

Length of yoke = length of arm = 21 inches

Allowing 20% for leakage flux from pole to arm

$$\varphi \text{ in yoke} = \frac{16.6 \times 1.2}{2} = 9950000 \text{ lines.}$$

$$\frac{9950000}{80000} = 124 \text{ square inches.}$$

$\frac{124}{21}$ = 5.4 inches radial depth of yoke.

Mean diameter = 45 - 3 - 16 = $\frac{26}{2}$ = 13 inches.

Mean length of path = $\frac{13 \times \Pi}{2 \times 6}$ + 13 = 16.4 inches.

POLE CORE AND FIELD.

Assume length of field core 10 inches.

From compounding curves at full load, 8 P. F. lag.

14400 ampere turns are required as will be seen later.

Assume 100 amp. in field, gives 144 turns per pole at 1500 amperes per square inch gives $\frac{100}{1500}$ = .0675 square inches.

Make ribbon for pole .75 inches x .09 inches wide - z layers.

Insulation between layers = .01 makes .1 inches total.

Length of coil = $\frac{144 \times .1}{2}$ = 7.2 inches.

Make coil 7.2 inches long cut pole to 8 inches in length make shoe 1.5 inches at center of pole core.

Assume 80000 lines per square inch in pole core.

$\frac{19900000}{80000}$ = 248 square inches cross section area.

Since coil is 1.5 inches thick make pole core 21 inches wide.

Then $\frac{248}{21}$ = 13.8 inches = length at right angles to shaft.

Mean length of turn = (13.8 + 1.5 + 21 + 1.5 + .5) 2 = 70.6 inches.

$\frac{70.6 \times 144}{12}$ = 850 feet = length of wire on one pole.

Resistance of one foot of wire .0675 square inches = .00052 at 60

Resistance of one field pole = 850 x .00052 = .443

Total resistance of 6 poles, 3 in series, two branches in parallel = 1.329.

With 220 volts exciter voltage allows $\frac{220}{1.33}$ = 165 amperes to flow.

To allow 100 amperes to flow, total resistance of field and rheo-

stat $= \frac{220}{100} = 2.2$

Resistance of rheostat at 100 amperes = 2.2 = .87

This makes 61% drop in field and 39% in rheostat.

HEATING IN FIELD.

$I^2 R$ under worst conditions $= 100^2 \times 1.33 = 133000$ watts

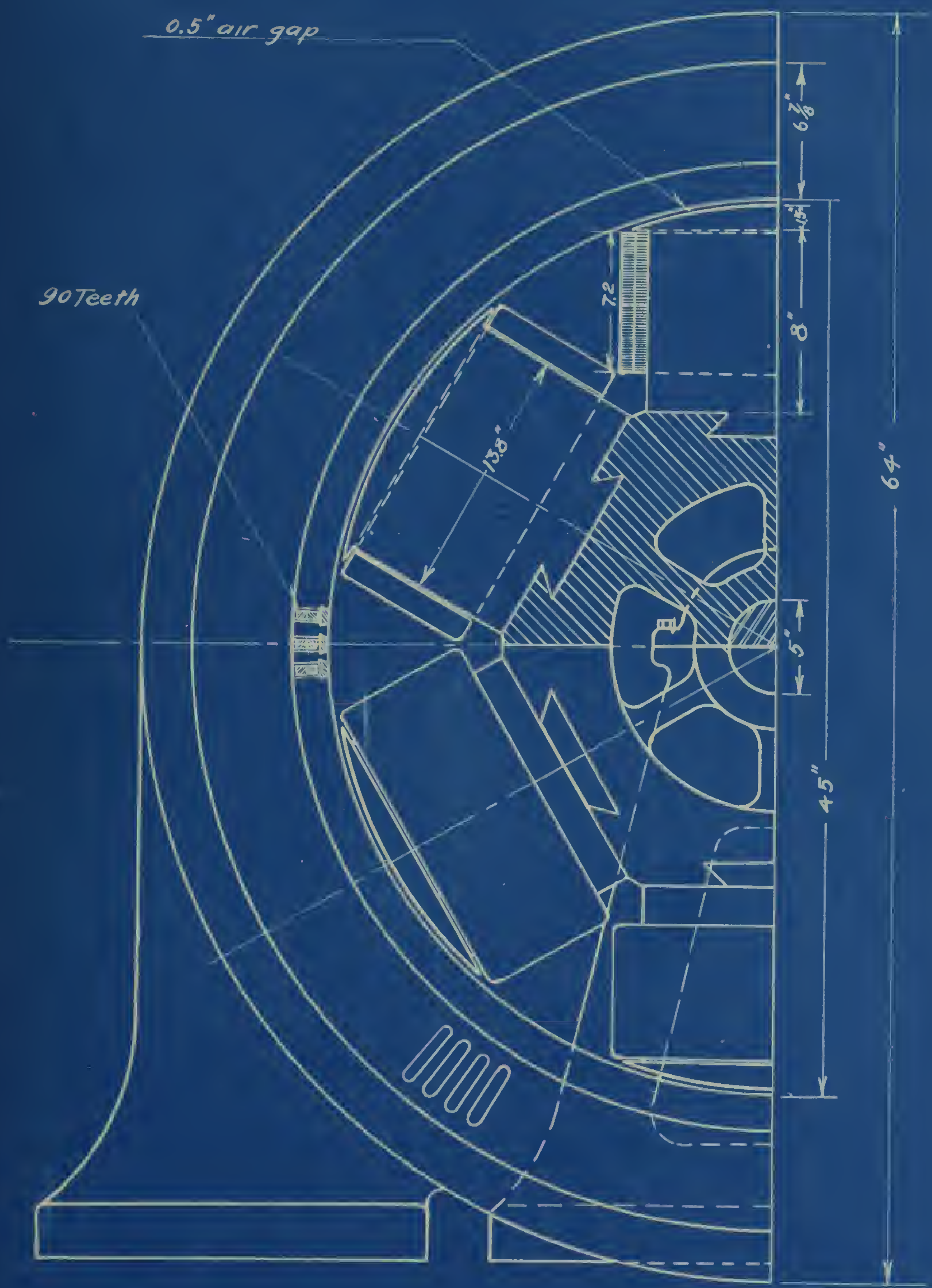
Outside radiating area = (13.8 + 3 + .5 + 21 + 3 + .5) 7.2 x 2 = 559 square inches.

Inside radiating area = (13.8 + 21 + .5 + .5) 7.2 x 2 = 472 square inches.

End area = (17.3 x 21.5) - (14.3 x 18.5) = 108 square inches.

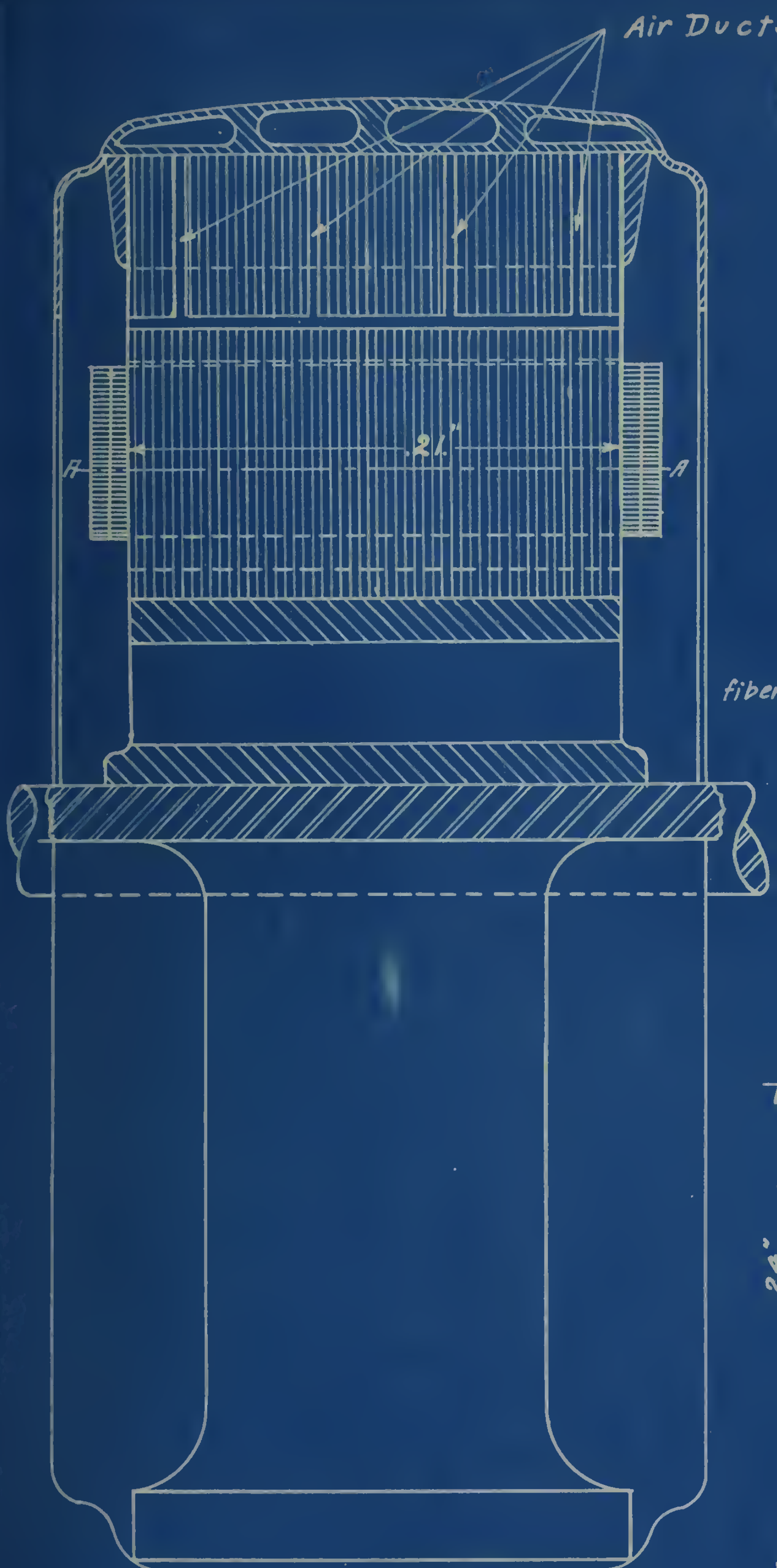
Radiating end area = 108 + 108 = 324 square inches.

Total radiating surface = 1355 square inches.

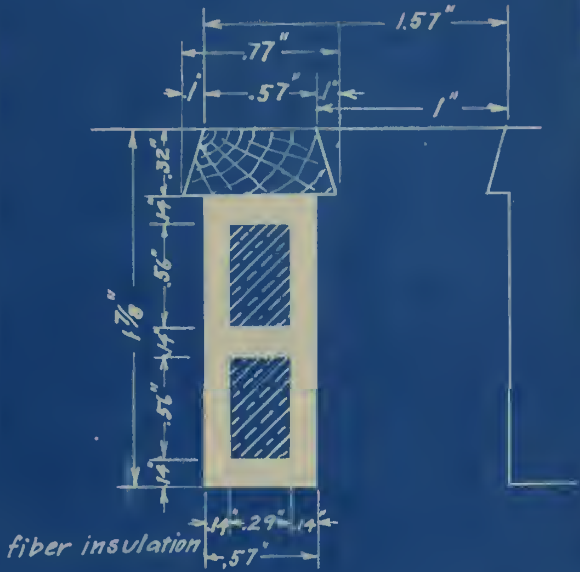


Scale $\frac{1}{8}$"=1"

Fig. 56 a

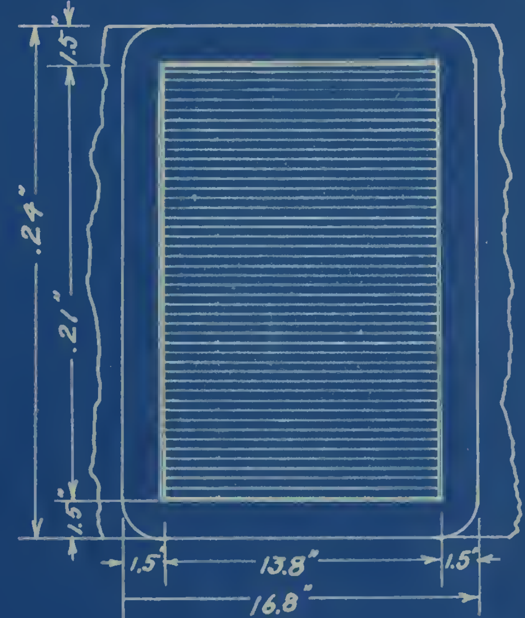


Scale 1/8" = 1"



Detail of Slot and Tooth
Scale-Actual Size

Cross Section thru AA



Field Coil. Two layers
Copper Ribbon - .09" X .75

Fig. 57a

SATURATION CURVES

Voltage 1330		Sq. in. area	megalines	A.T. per in.	length	Amp./ turn	Total A.T.
Armbody	80000	105	8.4	19.5	13.57"	264	
Pole core	80000	248	19.9	19.5	8"	156	
Air gap	65300	254	16.6		.5"	10000	
Yoke	80000	124	9.9	20.3	16.4"	334	
Teeth	100000	166	16.6	105	1 7/8"	197	10950

Voltage = 1800 between lines----1037 to neutral.

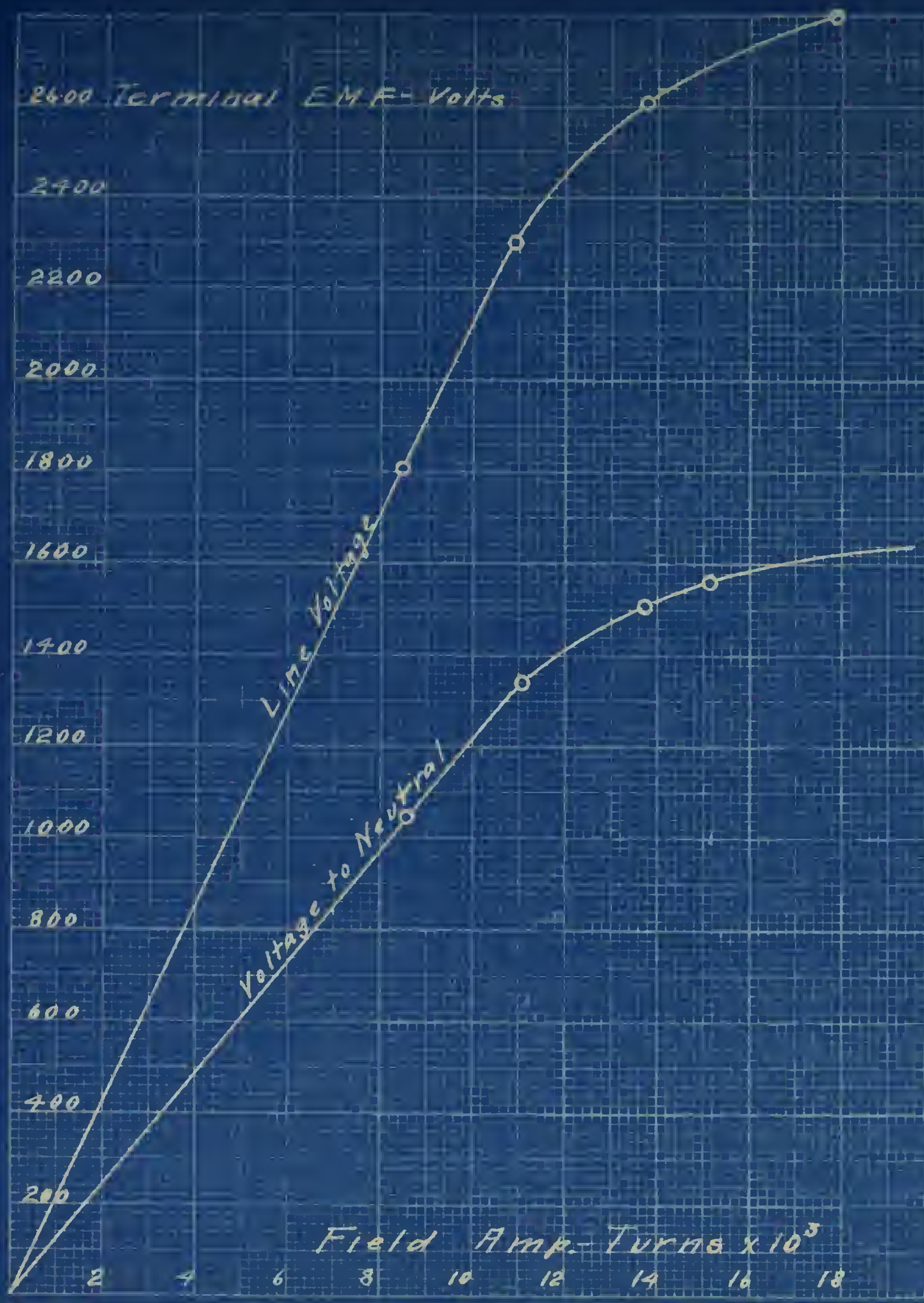
Armbody	61900	105	6.5	8.7	13.57	118	
Pole core	63000	248	15.6	8.9	8	71.1	
Air Gap	51200	254	13		.5	8010	
Yoke	63060	124	7.8	12.7	16.4	208	
Teeth	78500	166	13	18	1.875	34	8441

Vòltage = 1555 to neutral.

Armbody	92900	105	9.75	54	13.57	733	
Pole core	94500	248	23.4	63	8	504	
Air gap	76700	254	19.5			12000	
Yoke	94500	124	11.7	63	16.4	689	
Teeth	117500	166	19.5	445	1.875	835	15102

Voltage 1500 to neutral.

Armbody	84500	105	9.4	40	13.57	543	
Pole core	90000	248	22.25	42	8	336	
Air gap	74000	254	18.8		.5	11600	
Yoke	90000	124	11.13	42	16.4	689	
Teeth	113100	166	18.8	320	1.875	600	13768



Saturation Curves

Fig 58 a

PHASE CHARACTERISTICS

$$r = .013,\ x' = .06,\ x = .08,\ m' = \frac{4000}{377} = 10.61,\ m = 2/3\ 10.61=7.07$$

No Load

P. F.	.7 lag	.8	.9	1.0	.9 lead	8	7
i	0	0	0	0	0	0	0
i'	-384	-281	-183	0	183	281	384
e	1330	1330	1330	1330	1330	1330	1330
ir	0	0	0	0	0	0	0
i'x'	23.2	16.87	10.99	0	-10.99	-16.87	-23.2
e + ir - i'x'	1353.2	1346.8	1341	1330	1319	1313	1307
n	11250	11150	11100	10950	10900	10820	10790
i'r	4.99	-3.60	-3.30	0	3.38	3.66	4.99
i x	0	0	0	0	0	0	0
i'r + i x	4.99	-3.66	-3.38	0	3.38	3.66	4.99
n'	4.15	3.03	2.795	0	2.79	3.02	4.11
i m	0	0	0	0	0	0	0
i'm'	4070	2983	1940	0	-1940	-2983	-4070
A=i m + n'	4.15	3.03	2.8	0	3.38	3.63	4.99
B=n - i'm'	15320	14133	13040	10950	8960	7837	6720
F = $A^2 + B^2$	15320	14133	13040	10950	8960	7837	6720

PHASE CHARACTERISTICS

$r = .013,\ x' = .06,\ x = .08,\ m' = \frac{4000}{377} = 10.61,\ m = 2/3\ 10.61 = 7.07$

1/2 Load

P.F.	.7 lag	.8	.9	1.0	.9 lead	8	7
i	188.5	188.5	188.5	188.5	188.5	188.5	188.5
i'	-192	-140.5	-91.5	0	91.5	140.5	192
e	1330	1330	1330	1330	1330	1330	1330
ir	2.45	2.45	2.45	2.45	2.45	2.45	2.45
i'x'	11.6	8.44	5.49	0	-5.49	-8.44	-11.6
e + ir - i'x'	1344	1340.9	1338	1332.5	1327	1324	1320
n	11180	11100	11050	11000	10990	10950	10900
i'r	-2.49	-1.825	-1.19	0	1.19	1.825	2.49
i x	15.1	15.1	15.1	15.1	15.1	15.1	15.1
i'r + i x	12.61	13.275	13.91	15.1	16.29	16.9	17.59
n'	10.5	11	11.5	12.45	13.45	14	14.45
i m	1332	1332	1332	1332	1332	1332	1332
i'm'	-2040	-1489	-971	0	971	1489	2040
A=i m + n'	13425	1343	1343.5	1344.5	1345.5	1346	1346.5
B=n - i'm'	13220	12589	12021	11000	10019	9461	8860
$F = A^2 + B^2$	13270	12620	12100	11100	10100	9550	8960

PHASE CHARACTERISTICS

$$r = .013,\ x' = .06,\ x = .08,\ m' = \frac{4000}{377} = 10.61,\ m = 2/3\ 10.61 = 7.07$$

Full Load

P.F.	.7 lag	.8	.9	1.0	.9 lead	8	7
i	377	377	377	377	377	377	377
i'	-384	-281	-183	0	183	281	384
e	1330	1330	1330	1330	1330	1330	1330
ir	4.9	4.9	4.9	4.9	4.9	4.9	4.9
-i'x'	23.2	16.87	10.99	0	-10.99	-16.87	-23.2
e + ir - i'x'	1358.1	1351.7	1345.8	1334.9	1324	1317.1	1311.7
n	11250	11200	11100	11000	10900	10830	10800
i'r	-4.99	-3.66	-3.30	0	3.38	3.66	4.99
i x	22.6	22.6	22.6	22.6	22.6	22.6	22.6
i'r + i x	17.61	19.0	19.22	22.6	25.98	26.26	27.59
n'	14.6	15.75	15.9	18.65	21.4	21.6	22.6
i m	2665	2665	2665	2665	2665	2665	2665
i'm'	4070	2983	1940	0	-1940	-2983	-4070
A = i m + n'	2679	2680.2	2680.9	2683.6	2686.4	2686.6	2687.6
B = n - i'm'	15320	14183	13040	11000	8960	7847	6730
$F = A^2 + B^2$	15590	14400	13310	11320	9340	8390	7570

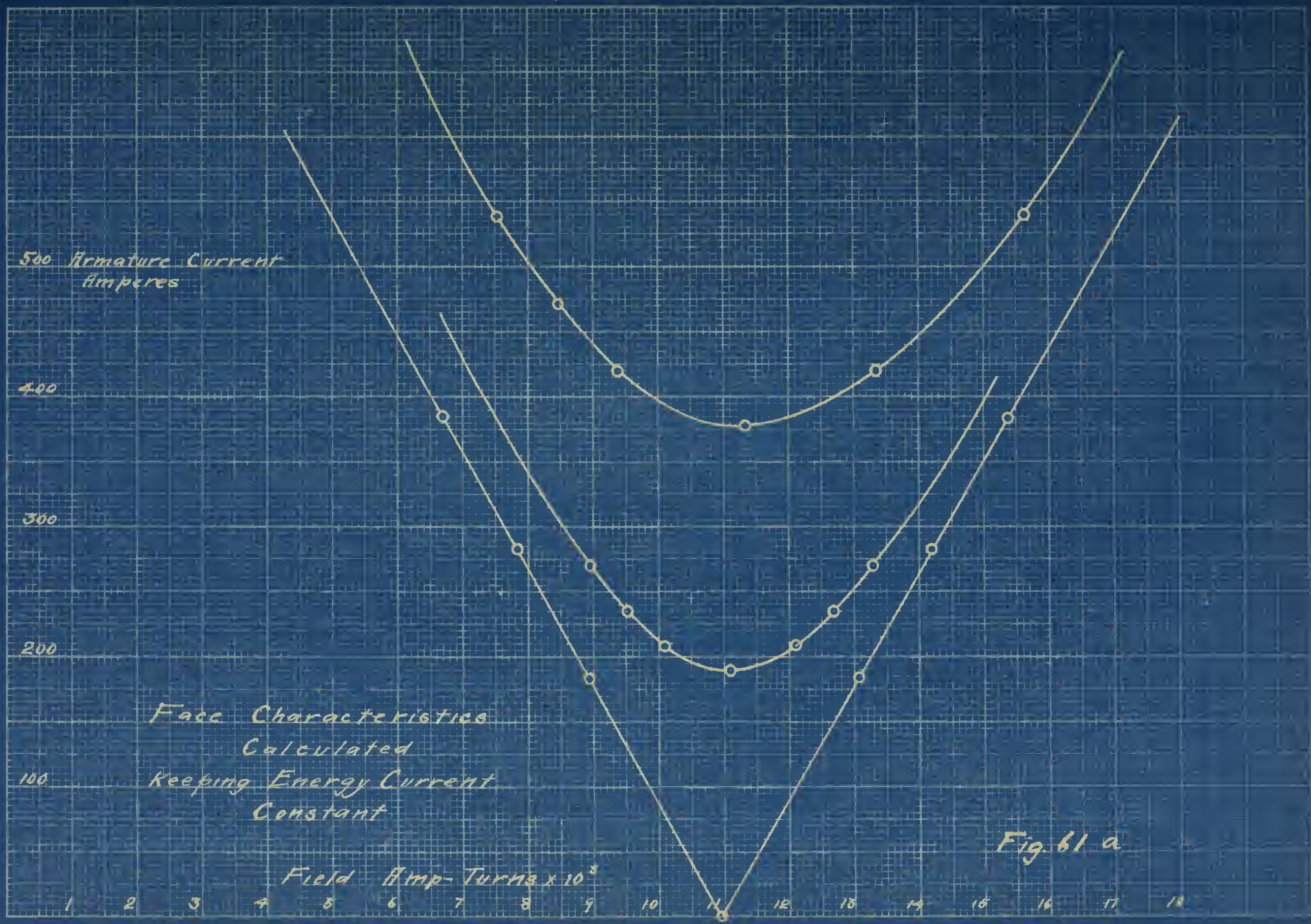

500 Armature Current
Amperes
400
300
200
100
Face Characteristics
Calculated
Keeping Energy Current
Constant
Field Amp-Turns x 10^3

Fig. 61 a

COMPOUNDING CURVES

Unity P.F.

Load %	25	50	75	100	125
I	94.3	188.5	282.8	377	471.3
i	94.3	188.5	282.8	377	471.3
i'	0	0	0	0	0
e	1330	1330	1330	1330	1330
i r	1.2	2.5	3.7	4.9	6.2
i'x'	0	0	0	0	0
e + i r - i'x'	1331	1332	1334	1335	1336
n	10950	10960	10970	10980	11000
i' r	0	0	0	0	0
i x	7.5	15.1	22.6	30.2	37.7
i' r + i x	7.5	15.1	22.6	30.2	37.7
n'	61.4	124	186	249	311
i m	666	1335	2000	2670	3340
i'm'	0	0	0	0	0
A = i m + n'	727.4	1459	2186	2919	3651
B = n - i'm'	10950	10960	10970	10980	11000
$F = A^2 + B^2$	11000	11040	11190	11350	11570

COMPOUNDING CURVES

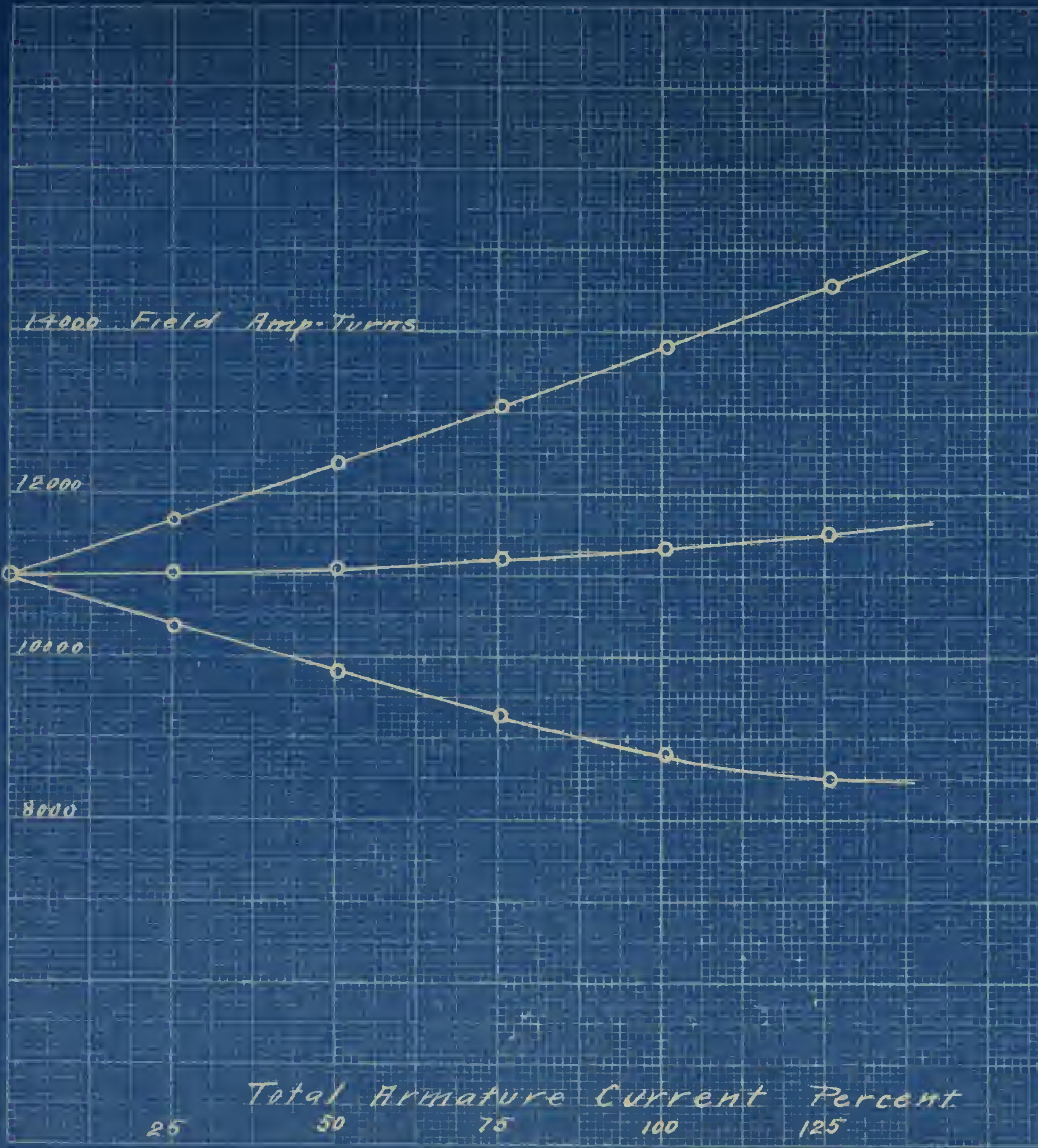
.8 Lag

Load %	25	50	75	100	125
I	94.3	188.5	282.8	377	471.3
i	75.5	150.9	227	301.8	377
i'	-56.5	-113	-170	-226	-283
e	1330	1330	1330	1330	1330
i r	.981	1.96	3.0	3.9	4.9
i'x'	-3.4	-6.3	-10.2	-13.6	-17
e + i r - i'x'	1335.4	1339.3	1343	1347.5	1352
n	11050	11100	11150	1120	11250
i' r	-.735	-1.5	-2.2	-2.8	-3.7
i x	6.05	12.1	18.2	24.1	30.2
i' r + i x	5.3	10.6	16	21.3	26.5
n'	44	88	145	228	287
i m	535	1069	1610	2140	2670
i'm'	-603	-1205	-1815	-2410	-3020
A = i m + n'	579	1157	1755	2368	2957
B = n - i'm'	11653	12305	12965	13610	14270
F = $A^2 + B^2$	11670	12380	13090	13810	14550

COMPOUNDING CURVES

.8 Lead

Load %	25	50	75	100	125
I	94.3	188.5	282.5	377	471.3
i	75.5	150.9	227	301.8	377
i'	56.5	113	170	226	283
e	1330	1330	1330	1330	1330
i r	.981	1.96	3.0	3.9	4.9
i'x'	3.4	6.8	10.2	13.6	17
e + i r - i'x'	1326	1325	1323	1320	1318
n	10930	10920	10910	10900	10880
i' r	.735	1.5	2.2	2.8	3.7
i x	6.05	12.1	18.2	24.1	30.2
i' r + i x	6.8	13.6	20.4	26.9	33.9
n'	56.2	112	168	216	280
i m	535	1069	1610	2140	3020
i'm'	60.3	1205	1815	2410	3020
A = i m + n'	591.2	1181	1378	2356	3300
B = n - i'm'	10327	9715	9095	8490	7860
$F = A^2 + B^2$	10350	9800	9250	8800	8520



Compounding Curves

Fig. 60 a

REGULATION

At full load, ampere turns = 13800 with .8 P. F. lag.

If switch is pulled voltage rises to 1510 volts as indicated from saturation curve.

$$\text{Reg.} = \frac{E - e}{e} = \frac{1510 - 1330}{1330} = 13.5\% \quad .8 \text{ Lag.}$$

Voltage at unity P. F. full load, if switch is pulled = 1360 volts.

$$\text{Reg.} = \frac{1360 - 1330}{1330} = 2.26\% \text{ Unity P.F.}$$

Voltage at .9 P. F. lead full load, if switch is pulled = 1095 v.

$$\text{Reg.} = \frac{1330 - 1095}{1330} = -17.6\% \quad .8 \text{ P. F. Lead.}$$

EFFICIENCY

Volume of iron in armature = mean diameter x π x depth x breadth.

Mean diameter at arm body = 22.5 + 1.875 + 2 1/2 = 26 7/8".

Volume of iron = (53.75 x π x 5(21 - 1.5)) .9 = 14800 cu. in.

Width of iron per cu. ft. = 495 lb.

Width of iron in arm. body = $\frac{14800}{1728}$ x 495 = 4250 lb.

Watts per lb. at 6000 - 800000 lines per sq. in. = 1.4 from curve.

Watts core loss in arm. body = 1.4 x 4250 = 5950 watts.

Volume of one tooth = .95 x (21 - 1.5) x 1.875 = 34.7 cu. in.

90 teeth gives $\frac{34.7 \times 90}{1728}$ = 1.81 cu. ft.

Watts per lb. at 6000 - 100000 lines per sq. in. = 2.36.

1.81 x 495 x 2.36 = 2120 watts.

Total core loss = 5450 + 2120 = 8070 watts = 8.01 K.W.

Friction loss = 1.5% = .015 x 1500 = 22.5 K.W.

Load loss = 1% = .01 x 1500 = 15 K.W.

I^2R loss of field = constant = 100^2 x 1.33 = 13300 watts.

EFFICIENCY - THREE PHASES

Load	I	K.W. Output	K.W. I^2R_a	K.W. I^2R_x	Load Loss in K. W.
25%	94.3	377	.346	13.3	15
50%	188.5	754.5	1.385	13.3	15
75%	282.5	1131.5	3.12	13.3	15
100%	377	1500	5.54	13.3	15
125%	471.3	1877	8.65	13.3	15

Load	K.W. Friction	K.W. Core loss	K.W. Total loss	K.W. Input	Percent Eff.
25%	22.5	8.07	59.25	436.25	86.5
50%	22.5	8.07	60.26	814.76	92.7
75%	22.5	8.07	62	1193.5	95
100%	22.5	8.07	64.41	1564.4	96
125%	22.5	8.07	67.52	1944.5	96

TORQUE ON SHORT CIRCUIT

$r = .013 \quad x' = .016 \quad z = .0614$ in percent $\quad r = .0037$

$x = .02 \quad z = .021.$

I in percent on short circuit $= \dfrac{E}{z} = \dfrac{1}{.021} = 47.6$

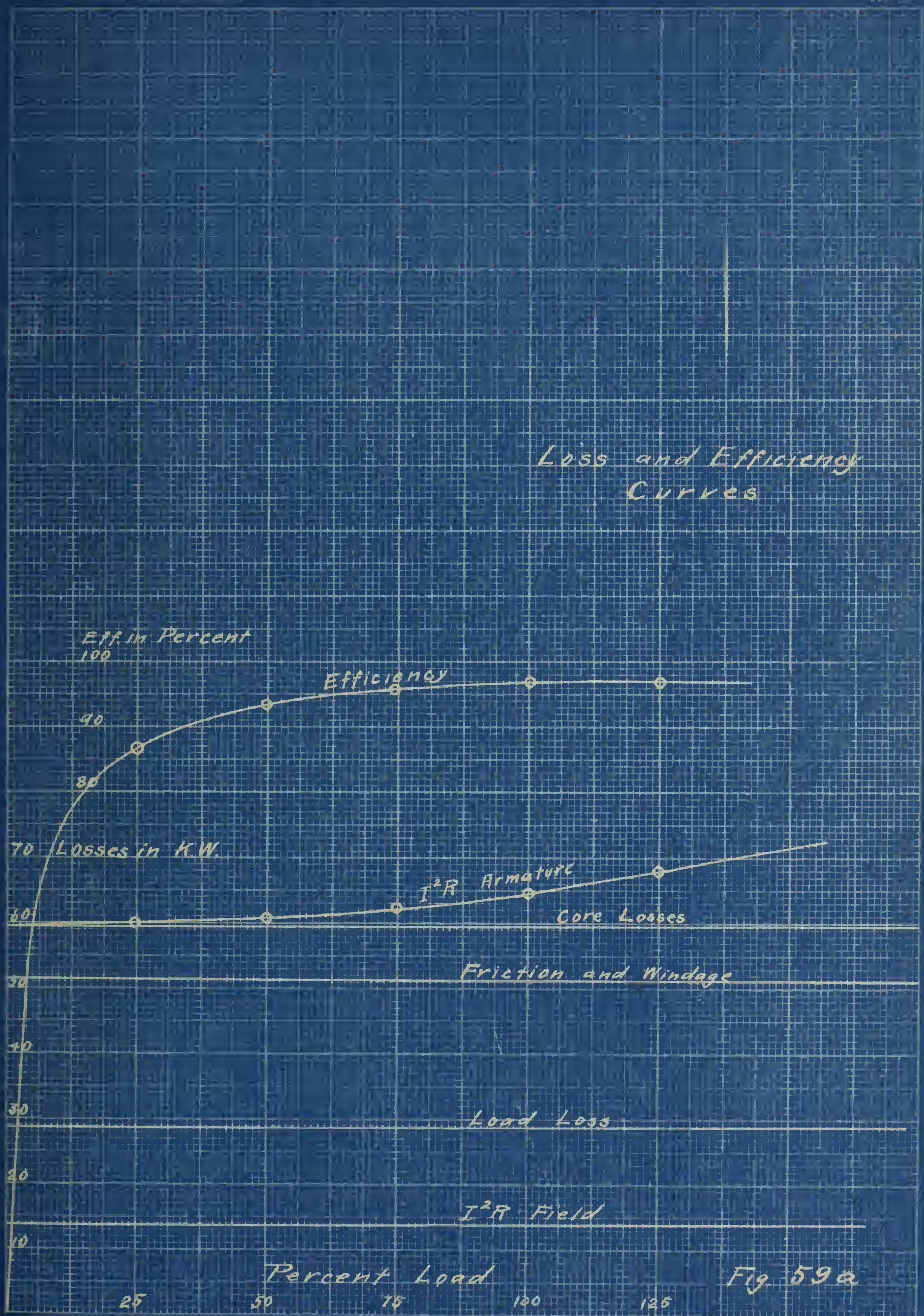
$$P = 3\ E\ I \cos$$

$$P.F. = \frac{.013}{.0614} = .212$$

$P = 3 \times 1 \times 47.6 \times 212 = 17.5$ times full load output.

If speed remains constant at instant of short circuit torque would be 17.5 times normal torque.

Therefore shaft must be 17 times size for normal torque.



Loss and Efficiency
Curves
Eff. in Percent
100
90
80
70
60
50
40
30
20
10
Efficiency
Losses in K.W.
I^2R Armature
Core Losses
Friction and Windage
Load Loss
I^2R Field
Percent Load
25
50
75
100
125

Fig. 59a

HEATING IN ARMATURE

Inside radiating area = $(21 - 1.5)\ 46 \times \pi = 2820$ sq. in.

Outside radiating area = $\frac{(21 - 1.5)(46 + 6\ 1/2)\pi}{2} = 1620$ sq. in.

Edge area = $\frac{2(46 + 3\ 7/16) \times 6\ 7/8}{1.5} = 453$ sq. in.

Area in air ducts = $\frac{8 \times (46 + 3\ 7/16)\ 6\ 2/8}{2} = 1365$ sq. in.

Total = 6258 sq. in.

Watts to be radiated at full load = core loss + I^2R_a + load loss =

$8.07 + 5.54 + 15 = 28.6$ K.W.

$\frac{28600}{6258} = 4.55$ watts per sq. in.

This is not to high since peripheral speed is 14000 feet per minute and the radiating of end turns is not considered.

IV M. M. F.

A magnet pole attracting or repelling another magnet pole of equal strength at unit distance with unit force is called a unit magnet pole, and the space surrounding a magnetic is called a magnetic field.

The magnetic field at unit distance from a unit magnet pole is called a unit magnetic field and its value is one line of force per sq. cm.

Hence from a unit magnetic pole thus issue a total of 4π lines of magnetic force. Since at unit radius in cms. from the pole in all directions the area is that of a sphere whose area is $4\pi r$ but since r = unity we have the total flux $\varphi = 4\pi$.

When an electric current flows in a conductor the space surrounding the conductor is a magnetic field, thus an electric current represents a magneto-motive force (m. m. f.)

The magnetic field of a straight conductor, whose return conductor is so far distant as not to affect the field, consists of lines of force surrounding the conductor in concentric circles.

The strength or intensity of the magnetic field is directly proportional to the current and inversely proportional to the distance from the conductor.

The magnetic circuit produced by an electric current surrounds the electric circuit, and inversely. That is the electric circuit and magnetic circuit are interlinked with each other.

Unit current in an electric circuit is that current which produces in a magnetic circuit of unit length (surrounding

the conductor) the field intensity 4π. That is produces as many lines of force per sq. cm. as issue from a unit magnet pole.

One tenth unit current is the practical unit or ampere. One ampere in an electric circuit or turn, that is, one ampere turn thus produces in a magnetic circuit of unit length the field intensity $\frac{4\pi}{10}$ and in a magnetic circuit of length L the field intensity $\frac{4\pi}{10\ L}$ and thus F ampere turns produce in a magnetic circuit of length L the field intensity $H = \frac{4\pi}{10\ L}$ lines of force per sq. cm. regardless of whether the F ampere turns are produced by F amperes in one turn or F turns through which one ampere flows, the magnetic circuit of course is in air.

In air we thus see that field intensity H and magnetic induction β are the same which gives us the general law for air.

(105) $\beta = \mu H$ Where μ is a measure of the permeability of the flux conducting medium and for air is taken as unity, while for some metals μ may reach a value as high as 6000.

Equation 105 may be written in several forms and for the sake of convenience it will be reduced to a more applicable form for design.

$$\text{For air } \beta = \mu H$$

$$= \mu \frac{.4\pi N I}{L} \qquad (106)$$

$$\frac{\beta L}{\mu\ 1.257} = N I \quad \text{but } \mu = 1 \text{ for air.}$$

$N I = .8 \beta L$ where β = lines per sq. cm., L = length of magnetic path in cms.

$$N I = \frac{.8 \beta}{\text{sq. cm.}} L \text{ cms.} \qquad (107)$$

But since 1 inch = 2.54 cms. and 1 sq. in. = 6.45 sq. cms. by substitution

$N\ I = .313\ \mathcal{B}\ L$ (108) where $\mathcal{B}$ = lines of force per sq. in. and L = length of magnetic path in inches. This value $N\ I$ corresponds to $\mathcal{B}$ maximum and I is the maximum value of current, to reduce to the effective value must be multiplied by $\frac{1}{\sqrt{2}}$. Hence the formula becomes for effective value of current

$$N\ I' = \frac{.313\,\mathcal{B}\ L}{\sqrt{2}} = .222\,\mathcal{B}\ L.$$

Where I' = the effective value of the current.

Again for a magnetic circuit in iron.

$\mathcal{B} = \mu H$ which reduces to $N\ I' = \frac{.313\,\mathcal{B}_{,}\ L\ 1\frac{1}{\mu}}{\sqrt{2}}$, and for a magnetic circuit of air and iron in series,

$$N\ I' = \frac{.313}{\sqrt{2}}\left[\mathcal{B}\ L + \frac{\mathcal{B}_{,}L_1}{\mu}\right] \qquad (109)$$

Where $N\ I'$ = ampere turns to produce the density $\mathcal{B}$ in lines per sq. in. in the air, density $\mathcal{B}_{,}$ in lines per sq. in. in the iron, where L = length of magnetic path in air, where L_1 = length of magnetic path in iron, μ = permeability of the iron at density $\mathcal{B}_{,}$.

By the introduction of the term reluctance R which by definition is equal to $\frac{1}{\mu}$ the equation for magnetic induction takes the form $\mathcal{B} = \frac{H}{R}$ which is very similar to the equation for the electric circuit $I = \frac{E}{R}$ and just as the total resistance of a number of electric circuits connected in series is the sum of their separate resistance, so if a circuit or a portion of a circuit is traversed by the same magnetic flux, the total magnetic reluctance is the sum of the separate magnetic reluctances, or

$$R = R_1 + R_2 + R_3 + \cdots$$

But unlike resistance, which is independent of the current density, reluctance is dependent upon the density of the flux passing a given section, that is the reluctance varies with the flux density for all magnetic materials.

Again if two or more magnetic circuits are so arranged as to have their magnetic paths in parallel, the total reluctance may be expressed in the same way as the total resistance of two or more electric circuits in parallel and for parallel reluctance. The total reluctance =

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \text{-----}} \qquad (110)$$

Consider the case of the shell type transformer, see figure 62 . The ampere turns of the winding mounted on the core will produce a flux which will follow the paths as indicated, and φ_1 will be equal to φ_2 provided the reluctance of the paths are equal. Again if the cross-section of the center core is twice that of each outside core there will be two paths of equal reluctance, R_1 and R_2.

$$\text{Now } \varphi = \frac{\text{m.m.f.}}{\text{Reluctance}} \qquad (110a)$$

$$\text{and} \quad R = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1\ R_2}{R_1 + R_2} \qquad (111)$$

$$\text{and} \quad \varphi = \text{m. m. f. } \frac{R_1 + R_2}{R_1\ R_2}\ .$$

It has been seen that when a current flows in an electric conductor, lines of force are produced which are interlinked with this conductor. If the current increases the number

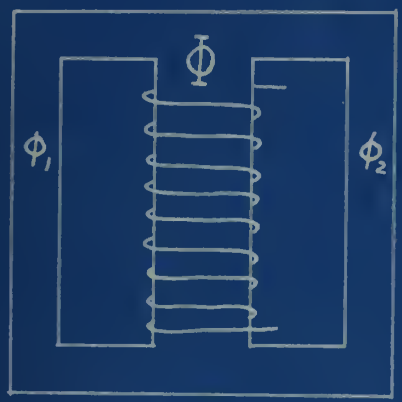


Fig. 62

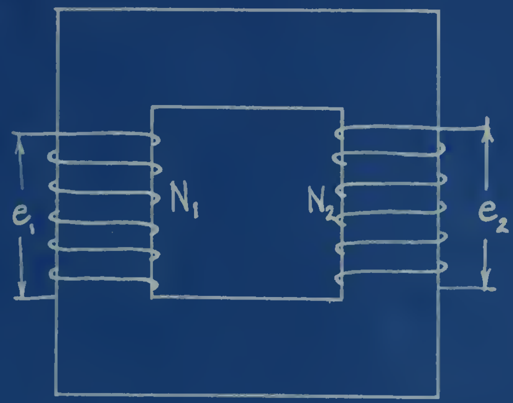


Fig. 62a

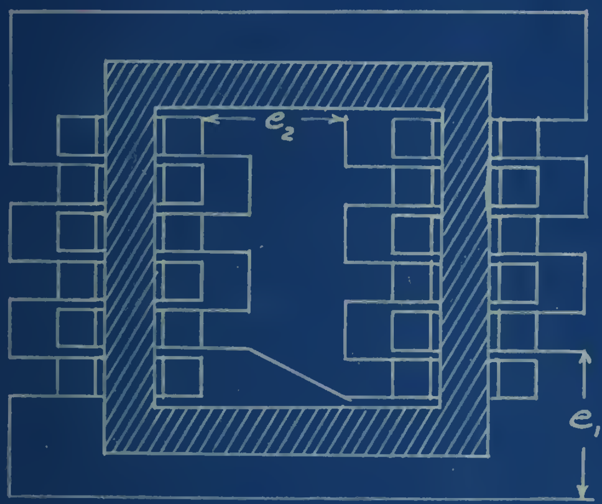


Fig. 63

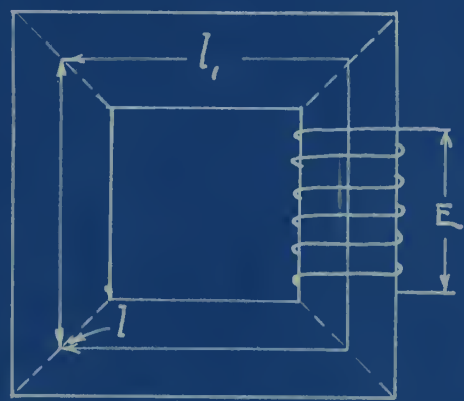


Fig. 63a

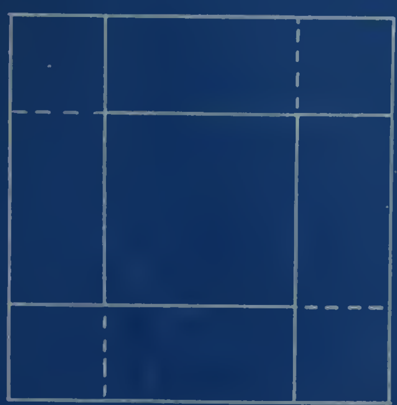


Fig. 64

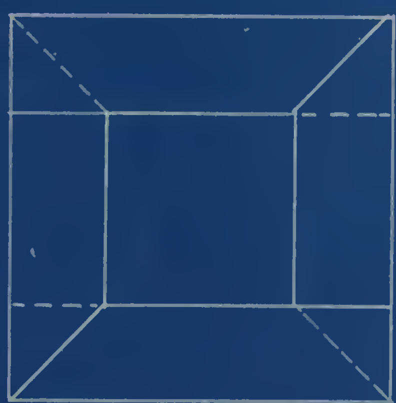


Fig. 65

of lines increase, if the current decrease the number of lines decrease. This is the condition for the generation of an electromotive force in the conductor.

If on a coil of wire a harmonically varying e. m. f. is impressed a current will flow therein. This current in turn will produce a magnetic flux which is alternating in value and of the same frequency as the impressed e. m. f. Since this flux passes from maximum in a given direction to zero and to a maximum in the opposite direction the electric conductor is cut by lines of magnetic force, this is the condition necessary for the generation of an e. m. f. which will be in opposition to the impressed e.m.f. and may be very nearly equal as by definition $e = n \frac{d\varphi}{dt}$, where e = e. m. f. at any instant corresponding to the cutting of $d\varphi$ lines of force in time dt by n conductors.

If φ is a harmonically varying quantity then,

$$\varphi = \underline{\varphi} \sin \omega t$$

where φ = value of the flux at any instant and $\underline{\varphi}$ = maximum value of φ and ωt = a uniformly varying angle.

Then
$$\frac{d\varphi}{dt} = \omega \underline{\varphi} \cos \omega t .$$

By substitution $e = N \omega \underline{\varphi} \cos \omega t$ and e will be a maximum when $\cos \omega t$ = unity.

Or
$$E_{max.} = N \omega \underline{\varphi} \text{ , but } \omega = 2 \pi f \text{ and}$$

$$E = N \underline{\varphi}\, 2 \pi f . \qquad (112)$$

Now in order to reduce to the effective value of e. m. f. which is equal to $\sqrt{\text{mean } e^2}$, find the relation between the maximum value of a sine wave of e. m. f. and its effective value.

Let $e = E \sin x$, then by definition

$$e_{eff.} = \sqrt{\int_0^{\pi} \frac{E^2 \sin^2 x \, dx}{\pi}} \tag{113}$$

$$= \sqrt{\frac{E^2}{\pi} \int_0^{\pi} \sin^2 x \, dx} \tag{114}$$

$$\sin^2 x = \frac{1}{2} - \frac{\cos 2x}{2}$$

$$\int_0^{\pi} \sin^2 x \, dx = \int_0^{\pi} \frac{dx}{2} - \frac{1}{2} \int_0^{\pi} \cos 2x \, dx \tag{115}$$

$$= \frac{\pi}{2} .$$

Substituting in 113 the value of $\int_0^{\pi} \sin^2 x \, dx$

$$e_{eff.} = \sqrt{\frac{E^2}{\pi} \cdot \frac{\pi}{2}} = \sqrt{\frac{E^2}{2}} = \sqrt{\frac{E^2}{2}} . \tag{116}$$

Hence the ratio of effective value to maximum value is $\frac{1}{\sqrt{2}}$ and reducing equation 112 to effective value,

$$e_{eff.} = \frac{N \underline{\varphi} \, 2 \pi f}{\sqrt{2}}$$

$$e_{eff.} = 4.44 \, f \, \underline{\varphi} \, N, \tag{117}$$

Where $\underline{\varphi}$ = maximum flux through coil.

N = number of turns interlinked with $\underline{\varphi}$.

f = frequency in cycles per second.

$e_{eff.}$ = effective e. m. f. in ab. volts, or $e_{eff.}$ in volts $= 4.44 \, f \, \underline{\varphi} \, N \times 10^{-8}$, which is the fundamental equation of the trans-

former, the application of which will be given later.

If two coils A and B could be so arranged that the flux produced, when a current flows in coil A would interlink coil B then an e. m. f. would be produced in coil B proportional to this flux, the rate of cutting and the number of turns therein, or if N_1 = turns in coil A, φ = flux produced when a current flows in coil A. See figure 62a.

Let N_2 = turns in coil B and let these turns be cut by the same flux φ.

$$\text{Then } e_1 = N_1 \frac{d\varphi_1}{dt}$$

$$e_2 = N_2 \frac{d\varphi_2}{dt}$$

or $\frac{e_1}{e_2} = \frac{N_1}{N_2}$ and the ratio $\frac{N_1}{N_2}$ is called the ratio of transformation.

This of course is the ideal case or the case where there is no loss of e. m. f. in coil A due to (I R) drop and where all the flux produced by coil A cuts coil B. In practice this condition can never by reached as there will always be a loss in e. m. f. due to I R drop when a current flows over a conductor and there will always be a certain amount of flux produced by the current in coil A that will cut coil A and not cut coil B, and under load conditions there will be a flux produced by coil B which will cut the conductors of coil B, but will not cut the conductors of coil A. This flux is called the leakage flux and will be discussed more fully under "leakage flux".

The correct relation between impressed e. m. f. primary induced e. m. f. and secondary induced e. m. f. is,

$$e \text{ impressed} = ir + e_1 ,$$

$$= ir + N \frac{d\varphi}{dt} .$$

And with no leakage,

$$\frac{e - ir}{e_2} = \frac{N_1}{N_2} .$$

In the actual transformer the leakage flux may be very much reduced by interleaving the coils as shown in figure 63 and most modern transformers are so built as to take advantage of this feature.

Let us assume again that the impressed e. m. f. = the counter e. m. f. and that the former follows the sine law.

$$e = N_1 \frac{d\varphi}{dt} = E \sin \omega t = \sqrt{2}\, E_1 \sin \omega t,$$

where E_1 = the effective value of e. m. f.

Then

$$d\varphi = \frac{\sqrt{2}\, E_1}{N_1} \sin \omega t \, dt$$

$$\varphi = \frac{\sqrt{2}\, e_{eff.}}{N_1} \sin \omega t \, dt = \frac{\sqrt{2}\, E_1}{\omega\, N_1} \sin\left(\omega t + \frac{\pi}{2}\right) .$$

The above equation shows that the flux also follows a sine law leading the induced e. m. f. by 90° .

At no load the transformer takes a certain power which is expended almost entirely on hysteresis and eddy current losses, because the copper losses are then so small that they may be disregarded. The no load power P_0 is then practically equal to the iron losses P_i or,

$P_0 = P_i = E\, I'_0 \cos \phi_0$, where I'_0 is the effective value of a sine current which has the same effective value as the actual magnetizing current I_0 and lags behind the e. m. f. by the angle φ_0.

$E\ I_0 \cos \varphi_0$ = hysteresis and eddy loss.

Consider now the case of a single coil wound on an iron core which corresponds to the primary of a transformer and let E = the effective value of the impressed e. m. f. assuming a sine wave of electro-motive force.

Then

$$\Phi = \frac{E\ 10^8}{4.44\ f\ N}. \qquad (118)$$

In other words when an effective e. m. f. = E is impressed on a coil of N turns the frequency being f cycles per second, there must be produced a flux in the iron core which inter-links with the N turns, whose maximum value Φ is,

$$\varphi = \frac{E\ 10^8}{4.44\ f\ N}$$

provided the I R drop in the coil is neglected and for practical purposes this is the case since at no load the current is usually about 2% of full load value and the I R drop is very small or we may say that the impressed e. m. f. is equal to the counter e. m. f.

The necessary ampere turns to produce this flux will consist of those wanted for the iron part of the circuit and those to force the flux through the air gaps or joints.

The ampere turns to produce the flux in the iron are obtained from a saturation curve of the particular iron used or by application of formula 109 provided the proper value of μ is known. This value of μ may be obtained from a μ - β curve for the iron under discussion.

From the saturation curve which is either a β - H or β - A T curve we see that the value of the magnetizing force depends on the value of β and since $\beta = \frac{\Phi}{\text{area}}$ it is evident that for a given maximum flux we will obtain a maximum flux density

with a given area. Therefore such an area of core in the transformer must be provided as will give a practical value of flux density. From a study of the saturation curve we also see that beyond a certain flux density the magnetizing force increases much more rapidly than the flux density and therefore we have the following condition. That is, the greater the flux density the greater the number of ampere turns to produce it but not in a direct proportion.

The greater the flux density the less the area of the core and hence for a given length of magnetic path the less the volume of iron and the shorter the length of one turn that is wound on the iron core. The greater the flux density the greater the hysteresis loss and eddy current loss as will be shown later.

The calculation of the A.T. to send the flux through the joints depends only on the density at the joints and the length and may be calculated directly by using equation 109 or,

$$\text{Effective A. T.} = \frac{.313}{\sqrt{2}} \beta \; L .$$

Or if β_1 = density in the iron in $\overline{\text{inches}}^2$,

L_1 = mean length of magnetic path in inches,

μ = value of permeability at density ,

L = length of the joints,

β = density at the joints,

Y = number of joints. See figure 63a.

Then A. T. effective to produce these densities is by equation

$$A.\ T. = \frac{.313}{\sqrt{2}} \left[\beta L Y + \frac{\beta_1 L_1}{\mu} \right]$$

or if $\beta = \beta_1$ which is usually the case,

$$A.\ T. = \frac{.313}{\sqrt{2}} \left[\beta L Y + \frac{L_1 \beta_1}{\mu} \right]$$

Hence by the application of formula 109 we may calculate the effective ampere turns for any transformer, when μ , L_1, Y , L and μ are known, μ of course may be determined from the saturation curve pretaining to the particular iron which we are using. L and L_1 being taken as the mean paths of the flux in the air and iron circuits. Figure 33 represents such an iron curve and figure 32 shows the watts lost in this particular iron due to eddy currents and hysteresis at various densities and at a frequency of 60 cycles. By using figure 32 we may calculate the total iron losses for any density at 60 cycles when we know the volume or weight of the iron core.

The waste of energy due to eddy current loss may be reduced by laminating the wire core and insulating these sheets with varnish or iron oxide. This process increases the length of the eddy current path and reduces its cross section thereby increasing the value of the resistance and decreasing the value of current.

Besides causing an I^2R loss the eddy current causes an uneven distribution of flux in the lamination since their demagnetizing action is greatest at the center. The calculation of the exact value of the eddy current is further complicated by the inductance in their paths. But by assuming the inductance as negligible we may obtain an approximate idea of their value and with what quantities they vary.

It has been found that after transformers have been worked for some time the iron losses are increased by a considerable amount while the permeability has at the same time been reduced. This phenomenon is called "ageing". Data has been presented by

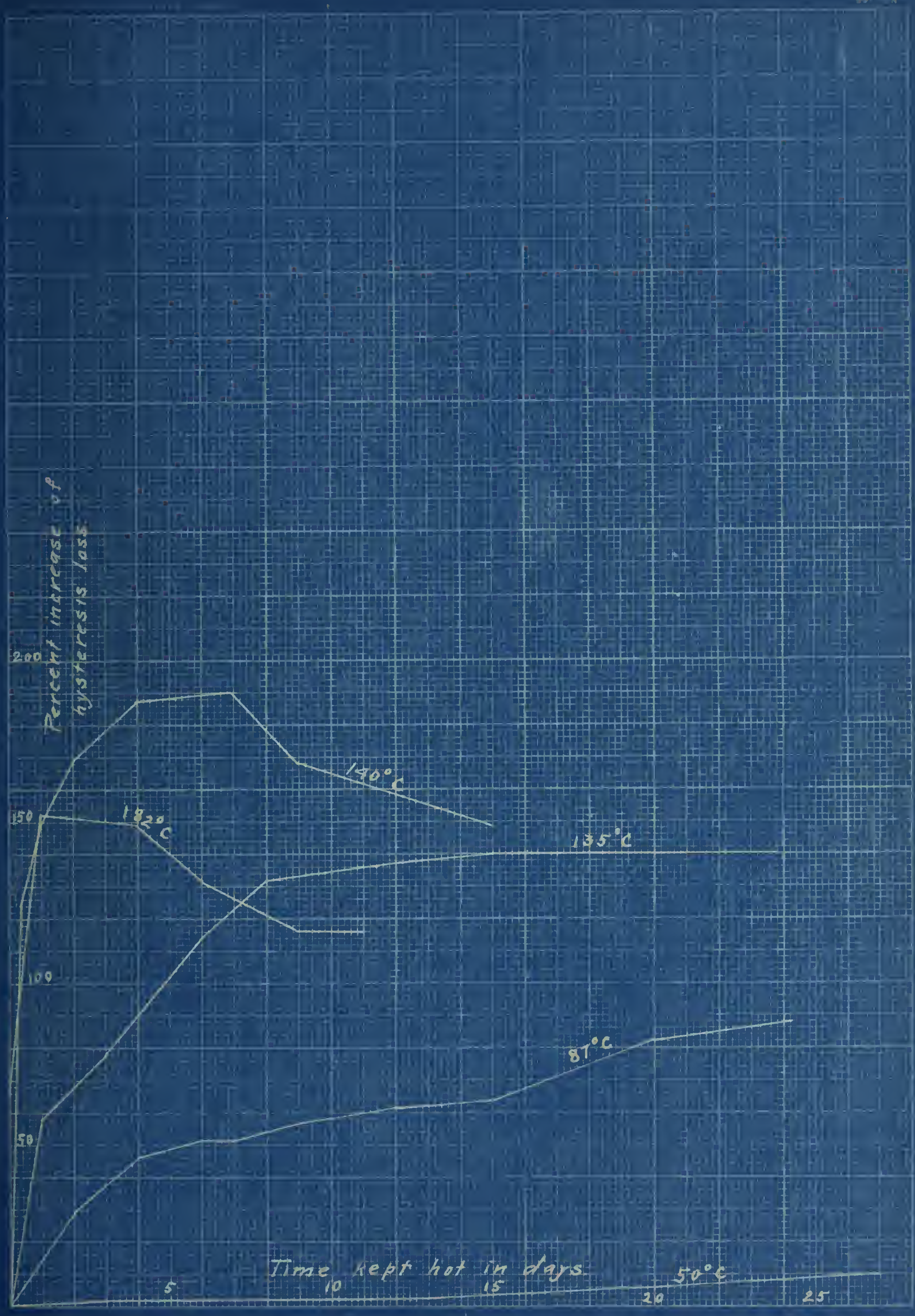
Mr. G. W. Partridge in the Electrician of December 7, 1894, showing that on three transformers the iron loss increased from 35% to 50% in the course of one to two hundred days. The following curves show data taken by others which illustrates the phenomenon more clearly. In the more recently produced transformer iron this phenomenon is almost absent as very little ageing will take place even at high temperatures and it is only very recently that iron has been produced which will show a very much lower hysteresis and eddy current loss than the values given in figure.

INFLUENCE OF JOINTS

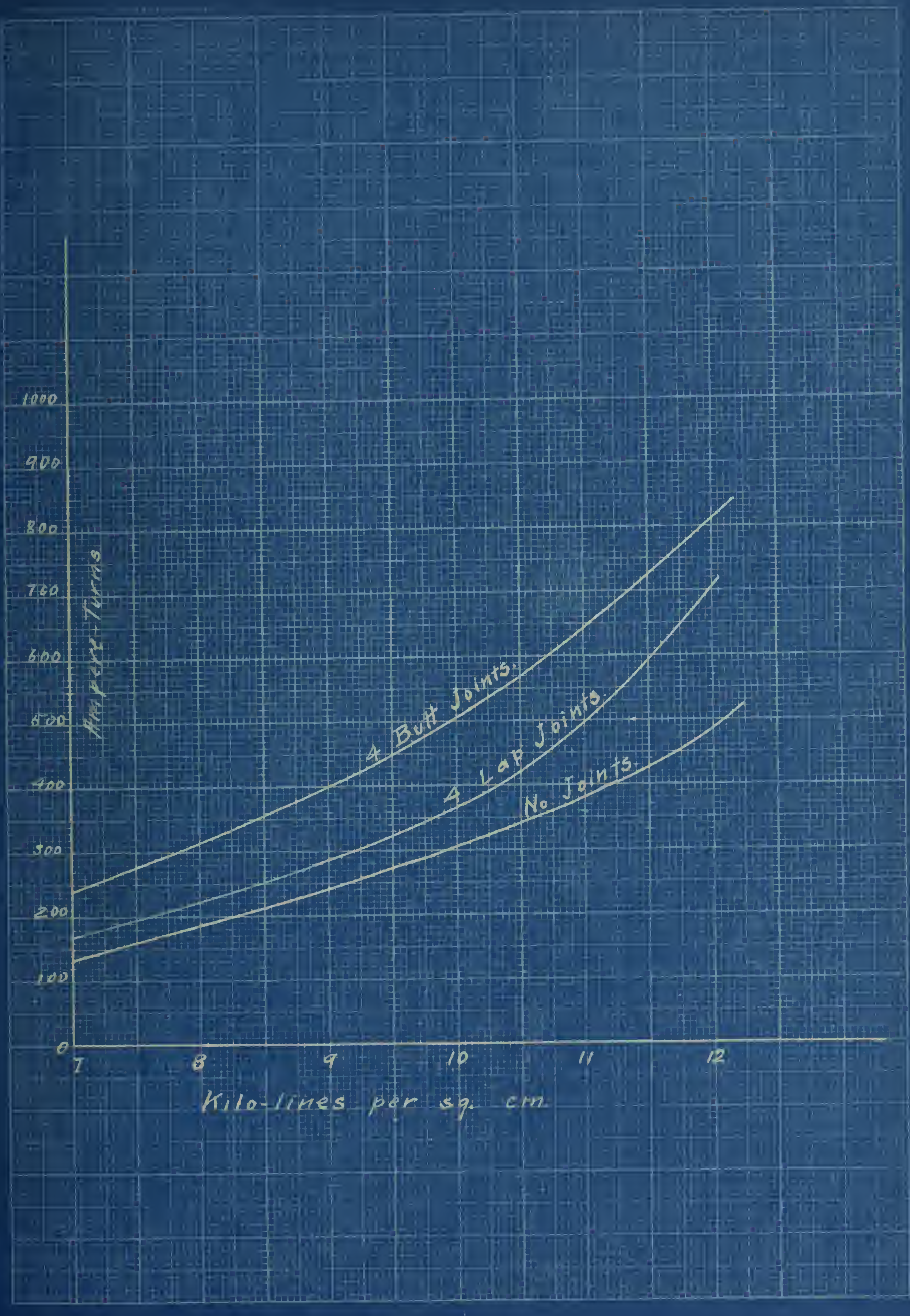
The magnetic circuit is usually built up in such a manner as to contain several joints. This is done to make it possible to use former wound coils since this type of coil is cheaper to manufacture, easier to insulate and mount in position on the core than the hand wound coil. The magnetic path is thus obstructed by joints which may be either of the butt or lap type.

From experiments by Ewing it is evident that the reluctance of a turned butt joint in a solid iron rod is about the same as that of an air gap .05 m.m. or 2 mils thick when there is practically no compression and less when the surfaces are forced together.

As the iron part of a transformer is built up of insulated sheets the joint will naturally be rough and even if planed off would not be as good as in solid metal. Hence a fair estimate of the length of joints with laminated surfaces is approximately 4 mils. The following curves give very good results for joints in a transformer core and may be used without great error.



Percent increase of hysteresis loss
200
150
100
50
140°C
182°C
135°C
87°C
50°C
Time kept hot in days
5
10
15
20
25



Ampere Turns
1000
900
800
700
600
500
400
300
200
100
0
7
8
9
10
11
12
Kilo-lines per sq. cm
4 Butt Joints.
4 Lap Joints.
No Joints.

The reluctance of the joints may be reduced by over lapping at the ends but we must insulate the plates in this case or the eddy currents become excessive.

Figures 64 and 65 show two methods of building up cores with joints over lapping. Figure 65 has the area of the air joint increased by cutting the lamination on a angle, thereby decreasing the density and hence the m. m. f. necessary to force the total flux through the magnetic circuit.

MAGNETIZING AND NO LOAD CURRENT

The magnetizing component of the no load current may be found by

$$I_m = \frac{X_i + X}{N_1}, \tag{119}$$

where X_i = ampere turns to send flux through iron circuit,

X = ampere turns to send flux through the air current,

N_1 = total turns on primary side of transformer.

In order to find I_c= (core loss current) at no load, we must know the iron losses at no load, or $P_i = P_h + P_e$. At no load the iron loss is practically the total power consumed, since the copper loss is very small and again if we make the assumption that at no load the induced e. m. f. is equal to the impressed e. m. f. we may write,

$$P_o = P_i = E\, I_c$$

$$\text{or} \quad I_c = \frac{P_i}{E}. \tag{120}$$

And therefore I_{ex} which is the vector sum of I_c and I_m=

$$I_{ex} = \sqrt{I^2_c + I^2_m} \tag{121}$$

See fig. 66.

$$= \sqrt{\frac{P_i^2}{E^2} + \left(\frac{X_i + X}{N_1}\right)^2} \quad . \qquad (122)$$

By previous equations we are able to calculate all the necessary data to determine I_{ex}. As the core loss in watts per pound at any density may be found from the curve sheets for the iron used. See curve sheet number 32 .

CALCULATION OF LEAKAGE REACTANCE

Let the leakage flux be divided into six parts φ_1, φ_2, φ_3, φ_4, φ_5, φ_6.

φ_1 = flux produced by primary and cutting through the primary coil itself.

φ_2 = flux produced by the primary and passing between primary and secondary in iron.

φ_3 = flux produced by primary and passing between the primary and secondary end turns.

φ_4 = flux produced by secondary and cutting through the secondary winding.

φ_5 = flux produced by secondary and passing between the primary and secondary in the iron part of the circuit.

φ_6 = flux produced by secondary and passing between the primary and secondary end turns. See fig. 67.

THEORY

The theory is based on the assumption that if φ lines cut 1/3 the primary turns it has the same effect as $\frac{\varphi}{3}$ lines cutting all the turns.

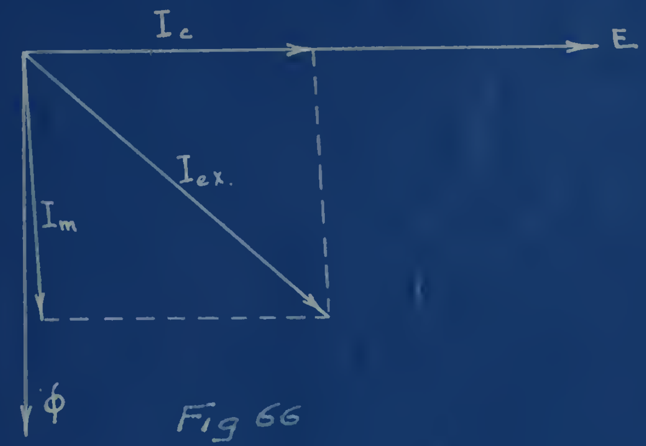


Fig 66

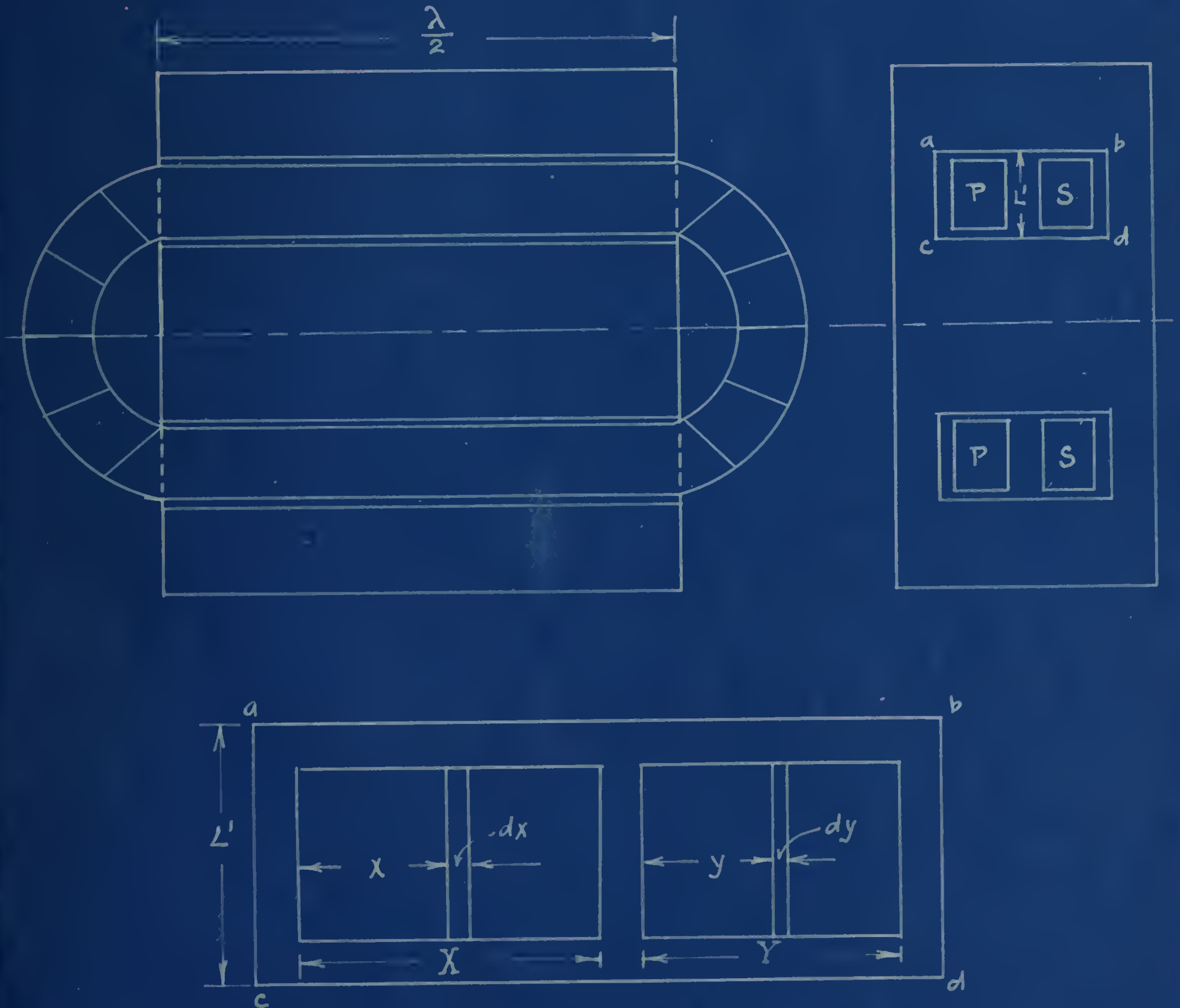


Fig 67.

First take the flux that passed through the section dx and call this flux $d\varphi$, then

$$d\varphi = \frac{\text{m. m. f.}}{\text{Rel.}} = \text{See equation (110a)} \frac{\dfrac{4 \pi N' i' x}{\overline{X}}}{\dfrac{L'}{dx\,\lambda}} = \frac{4 \pi N' i' x\, dx\, \lambda}{\overline{X} L'}, \text{ since m. m. f.} = 4 \pi N' i' \frac{x}{\overline{X}}$$

and the reluctance = length of $\frac{\text{path}}{\text{area}}$, but this flux only cuts $\frac{x}{\overline{X}}$ of the coil and hence has the effect of an equivalent flux $d\varphi_1$ =

$$\frac{4 \pi N' i' \lambda x^2\, dx}{\overline{X}^2 L'}, \text{ which cuts the entire coil.}$$

Therefore the equivalent φ_1 which cuts the entire coil is,

$$\varphi_1 = \int_0^{\overline{X}} \frac{4 \pi N' i' \lambda x^2 dx}{\overline{X} L'} = \frac{4}{3} \frac{\pi i' \lambda \overline{X} N'}{L'}.$$

In like manner

$$\varphi_4 = \frac{4}{3} \frac{\pi i'' N'' \lambda Y}{L'}.$$

But since $i' N' = i'' N''$,

$$\varphi_4 = \frac{4}{3} \frac{\pi i' N' \lambda Y}{L'}$$

$$\varphi_2 = \frac{4 \pi N' i'}{\dfrac{2 L'}{g \lambda}} = \frac{4 \pi N' i' g \lambda}{2 L'}$$

area = $\frac{g \lambda}{2}$ length = L'

Rel = $\frac{2 L'}{g \lambda}$

$$\varphi_5 = \frac{4 \pi N'' i''}{\dfrac{2 L'}{g \lambda}} = \frac{4 \pi N'' i'' g \lambda}{2 L'}$$

$$\varphi_5 = \frac{4 \pi N' i' g \lambda}{2 L'}$$

$$\varphi_3 = \frac{4 \pi N' i'}{\dfrac{2 L'}{g \rho}} = \frac{4 \pi N' i' g \rho}{2 L'}$$

$$\varphi_6 = \frac{4\,\pi\,N''\,i''}{\frac{2\,L'}{g\,\rho}} = \frac{4\,\pi\,N''\,i''\,g\,\rho}{2\,L'} = \frac{4\,\pi\,N'\,i'\,g\,\rho}{2\,L'}$$

$$\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4 + \varphi_5 + \varphi_6 = \underline{\varphi}$$

Total flux = $\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4 + \varphi_5 + \varphi_6$

$$\varphi_1 = \frac{4\,\pi\,N'}{3\,L'}\,i'\,\lambda\,\underline{\bar{x}}$$

$$\varphi_4 = \frac{4\,\pi\,N'}{3\,L'}\,i'\,\lambda\,y$$

$$\varphi_2 = \frac{4\,\pi\,N'\,i'\,g\,\lambda}{2\,L'}$$

$$\varphi_5 = \frac{4\,\pi\,N'\,i'\,g\,\bar{x}}{2\,L'}$$

$$\varphi_3 = \frac{4\,\pi\,N'\,i'\,g\,\rho}{2\,L'}$$

$$\varphi_6 = \frac{4\,\pi\,N'\,i'\,g\,\rho}{2\,L'}$$

Adding $\underline{\varphi} = \dfrac{4\,\pi\,N'\,i'}{L'}\left[\dfrac{\lambda\,\bar{x}}{3} + \dfrac{\lambda\,y}{3} + \lambda g + \rho\,g\right]$

But $L = \dfrac{N'\,\underline{\varphi}}{i'}$ or $\underline{\varphi} = \dfrac{L\,i'}{N'}$

$$\frac{L\,i'}{N'} = \frac{4\,\pi\,N'\,i'}{L'}\left[\frac{\lambda\,\bar{x}}{3} + \frac{\lambda\,y}{3} + \lambda g + \rho\,g\right]$$

$$L = \frac{4\,\pi\,N^2}{L'}\left[\frac{\lambda\,\bar{x}}{3} + \frac{\lambda\,y}{3} + \lambda g + \rho\,g\right] \qquad (123)$$

Where L is in cms. and x, y, λ, g, ρ, and L' are all in cms. to reduce L to henrys multiply by 10^{-9}.

The proper application of this equation will very

closely approximate the reactance of a transformer although care must be used in its application since with transformers composed of many coils it is often very difficult to determine the correct leakage path for the various sections.

Hobart has given the following equation for calculating the transformer reactance.

$$\%\ \text{reactance drop} = f\,\frac{a}{b \times c}\,, \qquad (124)$$

where a = effective value of primary ampere turns,

b = height of the winding space in cms.

c = core density in kilo-lines per sq. cm.

The curve in figure 69 gives values of f depending on the ratio of width of winding space to height of winding space. In using this formula care must also be taken since with transformers with a large number of coils this formula may lead to results which are very much too large.

The resistance and weight of the windings depends upon the following conditions.

(a) Number of turns to be used.

(b) Perimeter and hence the shape of the core.

(c) Amount of insulation used.

(d) Space between core and coils, between coils of secondary and primary.

(e) Current density to be used.

(a) The number of turns to be used on the transformer is a very important consideration since from the equation $E = 4.44 \times f \times \Phi \times N \times 10^{-8}$, it is seen that for a given frequency and e. m. f. there is the relation $\Phi = \frac{K}{N}$. We find the follow-

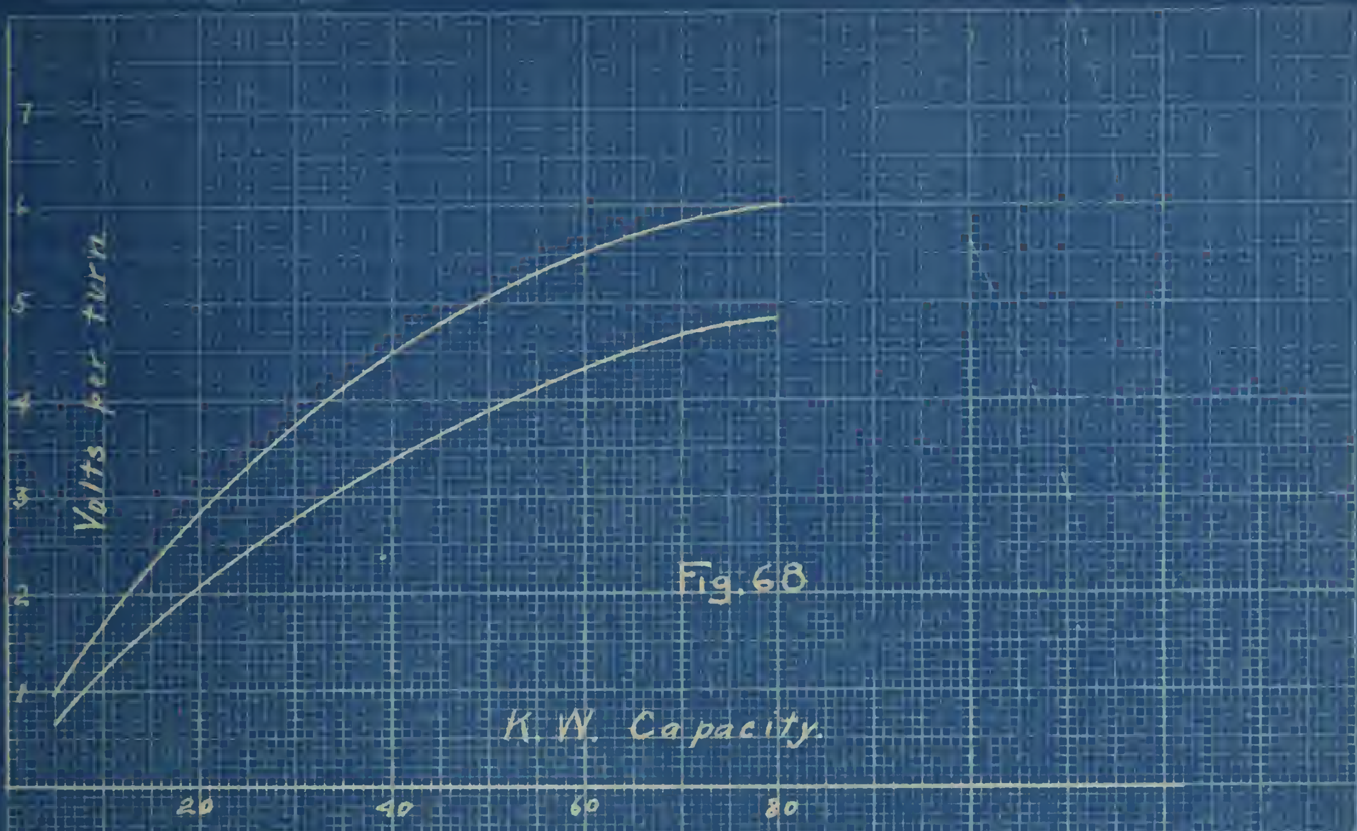

Volts per turn
1
2
3
4
5
6
7
K.W. Capacity.
20
40
60
80

Fig. 68

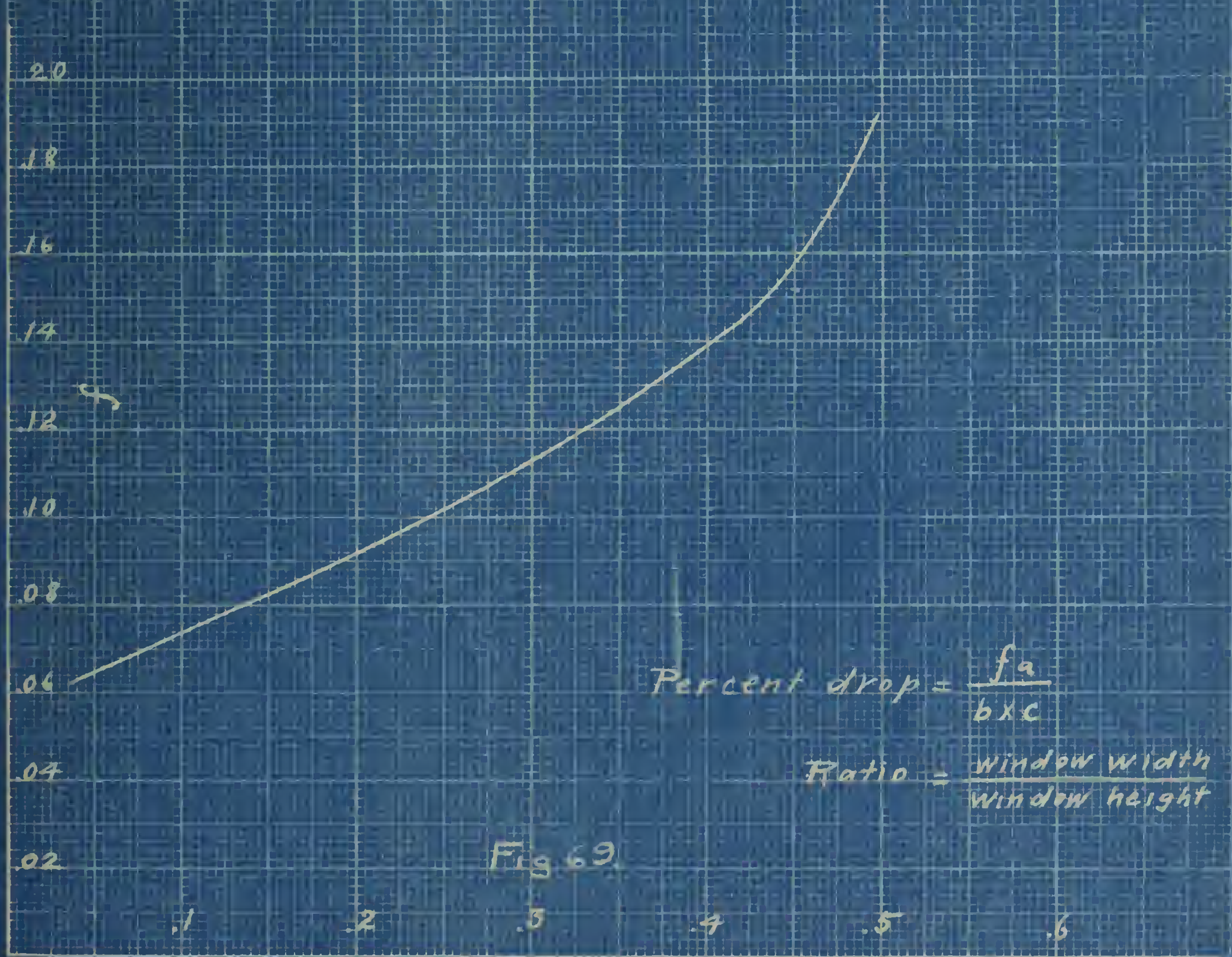

f
.02
.04
.06
.08
.10
.12
.14
.16
.18
.20
.1
.2
.3
.4
.5
.6
Percent drop = $\frac{fa}{b \times c}$
Ratio = $\frac{\text{window width}}{\text{window height}}$

Fig. 69.

ing condition,a large number of turns means a small amount of flux and vise-versa, the problem being to choose those turns that will give a maximum efficiency and in small transformers a minimum regulation with a small outlay for copper and iron.

This problem will for a given design require considerable attention. The curve figure 68 will give approximate or trial values of turns to be used for 25 and 60 cycle transformers. Conditions (b), (c) and (d) only enter in that the mean length of turn will be affected by changing any of these conditions. Transformers with long thin coils have the advantage of a shorter mean turn.

Condition (e) of course will affect the resistance and weight of copper very materially since the current density fixes at once the size of wire to be used and this of course fixes its resistance per unit length. The current density varies from 900 to 1300 amperes per square inch depending somewhat upon the shape of the coils.

The % primary resistance is usually somewhat greater than the % secondary since the low tension side is wound next to the iron core and with the same current density will have a lower % resistance due to the shorter mean length of turn. This however is not always the case as the secondary may have a higher current density and thus a % resistance equal to that of the primary. The determining factor in either case will be the shape of the coil and the watts per square inch that it must radiate.

INSULATIONS

Porcelain is a fire proof material possessing high

dielectric strength, but because of its brittleness it is only suitable for such purposes as bushings, connection block, fuse handles etc.

This material finds its application in transformer design as brushes for bringing both high and low tention leads through the transformer case and also as a connection block on the high tension side of lighting transformers. Other insulations have been discussed in the design of direct current machines.

LOSSES

The I^2R losses of the transformer are found in the following manner.

(a) Calculate mean length of turn for both primary and secondary.

(b) From the cross section area and the total length of wire the primary and secondary resistance may be determined.

(c) For any load the value of I_p and I_s may be determined and hence the I^2R losses for all loads calculated.

The core losses are found by the aid of curve sheet number 32 the only calculations being made are the flux density in the iron core and the weight of the iron.

Knowing these two losses the efficiency may be determined for any load since the voltage and core loss are assumed constant and the copper loss varies as the square of the load on the transformer.

The following discussion shows the complex quantity method of determining the regulation efficiency power factor etc. of the transformer with various types of loading.

CALCULATION OF THE REGULATION OF A TRANSFORMER

Object, to determine the regulation of a transformer by calculation from calculated constants.

Theory and Method, the open circuit core loss and magnetizing current have been obtained as previously shown, the reactance drop in primary and secondary may be found by application of equation 123. From these values and the resistance of the windings it is possible to calculate the regulation of the transformer under different conditions of loading.

In calculation of regulation it is convenient to express all quantities in percentage values. The following should be noted,

R primary in % = resistance drop in secondary due to full load current divided by primary voltage,

R secondary in % = resistance drop in secondary due to full load current.

X primary in % = primary reactance drop at full load current, divided by primary voltage.

X secondary in % = secondary reactance drop divided by secondary voltage.

a_5 (conductance in %) = the energy component of the exciting current in %.

b_5 (susceptance in %) = the magnetizing component of the exciting current in %.

By energy current is meant the component of the exciting current in phase with the electro motive force and by magnetizing current the component at right angles with the electro motive force. From the above it is seen that

the energy current is evidently the core loss watts divided by the voltmeter reading. The magnetizing current is derived from the exciting current. These values should then be expressed in percent of the secondary full load current. In this form they may be used as secondary conductance and susceptance if the secondary receiving voltage is taken as unity. The following equations show the development of the equations in complex quantities. A problem is also appended showing the application.

In figure 70, 1, 2, 3, represent any type of load; e is the voltage across this load, assumed constant. r_4 and x_4 are the secondary resistance and reactance; r_5 and x_5 together represent the impedance offered by the transformer on open circuit. r_6 and x_6 the primary resistance and reactance. E_o is the voltage impressed on the primary.

Hence $\frac{E_o - e}{e}$ is the regulation.

The following equations may be developed:

$$\dot{Z}_1 = r_1 + jx_1; \quad \dot{Z}_2 = r_2 + jx_2; \quad \dot{Z}_3 = r_3 + jx_3$$

Hence,

$$\dot{Y}_1 = \frac{1}{\dot{Z}_1} = \frac{1}{r_1 + jx_1} = \frac{r_1 - jx_1}{r^2_1 + x^2_1} = a_1 + jb_1$$

$$\dot{Y}_2 = \frac{1}{\dot{Z}_2} = \frac{1}{r_2 + jx_2} = \frac{r_2 - jx_2}{r^2_2 + jx^2_2} = a_2 + jb_2$$

$$\dot{Y}_3 = \frac{1}{\dot{Z}_3} = \frac{1}{r_3 + jx_3} = \frac{r_3 - jx_3}{r^2_3 + jx^2_3} = a_3 + jb_3$$

$$a_1 = \frac{r_1}{r^2_1 + x^2_1}; \quad b = \frac{-x_1}{r^2_1 + x^2_1}$$

a_2 ; b_2 ; a_3 ; and b_3 same as a_1 and b_1 except subscripts

or

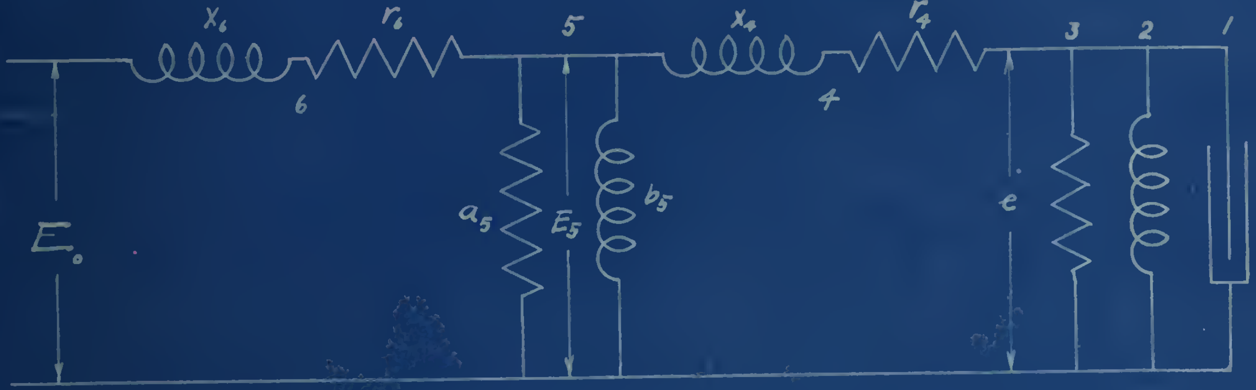


Fig. 70

$$\dot{Y} = (a_1 + a_2 + a_3) + j\,(b_1 + b_2 + b_3)$$
$$= A + jB$$

Where $A = a_1 + a_2 + a_3$,

$B = b_1 + b_2 + b_3$.

The current flowing in part 4 is then,

$$\dot{I}_4 = e\,(A + jB).$$

or numerically,

$$I = e\sqrt{A^2 + B^2}$$

and the power factor of the load is

$$P = \frac{A}{\sqrt{A^2 + B^2}}$$

The secondary impedance is expressed by,

$$\dot{Z}_4 = r_4 + jx_4$$

and the secondary impedance drop by

$$\dot{I}_4\,\dot{Z}_4 = e\,(A + jB)\,(r_4 + jx_4)$$
$$= e((Ar_4 - Bx_4) + j\,(Ax_4 + Br_4))$$

Hence the voltage induced in the transformer must be

$$\dot{E}_5 = e + \dot{I}_4\,\dot{Z}_4$$
$$= e\,((1 + Ar_4 - Bx_4) + j\,(Ax_4 + Br_4))$$

or $= e\,(C + jD)$.

where $C = 1 + Ar_4 - Bx_4$,

$D = Ax_4 + Br_4$ -

The admittance of part 5 representing the energy and magnetizing component of the exciting current is then,

$$\dot{Y}_5 = a_5 + jb_5$$

hence,

$$I_5 = e(C + jD)\,(a_5 + jb_5)$$

$$= e\ ((a_5 C - b_5 D) + j\ (b_5 C + a_5 D))$$

$$= e\ (F + jG)$$

where $F = a_5 C - b_5 D$

$G = b_5 C + a_5 D$

The current in the primary is then,

$$I_6 = I_4 + I_5,$$
$$= e\ (A + jB) + e\ (F + jG),$$
$$= e\ ((A + F) + j\ (B + G),$$
$$= e\ (H + jK).$$

where $H = A + F,$

$K = B + G,$

Since $Z_6 = r_6 + jx_6$,

the drop due to primary impedance is,

$$I_6\ Z_6 = e\ (H + jK)(r_6 + jx_6)$$
$$= e\ (Hx_6 - Kx_6) + j\ )Hx_6 + Kr_6))$$
$$= e\ (L + jM),$$

where $L = Hr_6 - Kx_6,$

$M = Hx_6 + Kr_6.$

Hence the impressed voltage is,

$$E_o = E_5 + I_6\ Z_6$$
$$= e\ (C + jD) + e\ (L + jM)$$
$$= e\ (C + L) + j\ (D + M)$$
$$= e\ (N + jO)$$

where $N = C + L,$

$C = D + M.$

Numerically $E_o = e\sqrt{N^2 + O^2}$

When e is taken as unity,

$$E_o = \sqrt{N^2 + O^2}$$

To illustrate the application of this method of calculation the following example is taken from a transformer actually installed for operation.

100 KW, 15,000 volts primary, 2350 volts secondary, 60 cycles. 6.67 amperes primary, 42.5 amperes secondary at full load.

The experimental data for this transformer gives the following:

Core loss, 1220 watts at 2550 volts.

Exciting current, 0.725 amperes at 2350 volts.

Impedance watts 1200 at 6.67 amperes.

Impedance volts 495 at 6.67 amperes.

Resistance of primary 17.96 ohms hot.

Resistance of secondary .2103 ohms hot.

The following calculations will give the values of the necessary constants, considering full load current and receiving voltage as 100%.

Resistance,

$$\text{Primary } 6.67 \times 17.98 = 119.5$$
$$119.5 = 0.8\% \text{ of } 15{,}000.$$
$$\text{Secondary } 42.5 = 0.2103 = 8.94$$
$$8.94 = .38\% \text{ of } 2350.$$

hence

$$r_6 = 0.008, \qquad r_4 = 0.0038.$$

Open circuit test,

$$\text{Energy current} = \frac{1220}{2350} = 0.522 \text{ amperes},$$

$$0.522 = 1.23\% \text{ of } 42.5 \text{ amperes.}$$

$$\text{Magnetizing current} = \sqrt{.735^2 - .522^2} = 0.505 \text{ amperes},$$

$$0.505 = 1.20\% \text{ of } 42.5 \text{ amperes.}$$

hence

$$a_5 = 0.0123, \qquad b_5 = -0.0120$$

Impedance test,

$$\text{Energy drop} = \frac{1200}{6.67} = 180,$$

180 - 1.2% of 15,000

Impedance drop = 495.

Reactance drop $= \sqrt{495^2 - 180^2} = 460.$

460 = 3.08% of 15,000.

Hence

$$x_4 = x_6 = 0.0154.$$

All that remains to complete the calculations is the assumption of a given load.

	Non-inductive	Inductive	Capacity
$P =$	1	0.8	0.8
$r_1 =$	---	---	0.8
$x_1 =$	---	---	-0.6
$r_2 =$	---	0.8	---
$x_2 =$	---	+0.6	---
$r_3 =$	1	---	---
$x_3 =$	---	---	---
$a_1 = \frac{r_1}{r_1^2 + x_1^2}$	---	---	0.8
$+b_1 = \frac{-x_1}{r_1^2 + x_1^2}$	---	---	+0.6
$a_2 = \frac{r_2}{r_2^2 + x_2^2}$	---	0.8	---
$b_2 = \frac{-x_2}{r_2^2 + x_2^2}$	---	-0.6	---
$a_3 = \frac{1}{r_3}$	1	---	---
$b_3 =$ --	0	---	---
$A = a_1 + a_2 + a_3$	1	0.8	0.8
$B = b_2 + b_3 + b_1$	0	-0.6	+0.6
$I_4 = \sqrt{(A^2 + B^2)}$	1	1	1

	Non-inductive	Inductive	Capacity
$e =$	1	1	1
$r_4 =$	0.0038	0.0058	0.0038
$x_4 =$	0.0154	0.0154	0.0154
$C = 1 + Ar_4 - Bx_4$	1.0039	1.0123	0.9938
$D = Ax_4 + Br_4$	0.0134	0.01004	0.0146
$a_5 =$	0.0123	0.0125	0.0125
$b_5 =$	-0.0120	-0.0120	-0.0120
$F = a_5C - b_5D$	0.0125	0.01257	0.01259
$G = b_5C + a_5D$	-0.01186	-0.01205	-0.01175
$H = A + F$	1.0125	0.81257	0.8124
$K = B + G$	-0.01186	-0.61203	+0.58823
$r_6 =$	0.008	0.008	0.008
$x_6 =$	0.0154	0.0154	0.0154
$L = Hr_6 - Kx_6$	0.00829	0.01592	-0.00255
$M = Hx_6 + Kr_6$	0.0155	0.0076	0.0172
$N = C + L$	1.0121	1.0282	0.9913
$O = D + M$	0.0309	0.0176	0.0318
$E_o = e\sqrt{N^2 + O^2}$	1.0126	1.0283	0.9918
Regulation	1.26%	2.83%	-.32%

A very convenient check on the final results may be had in the methods for obtaining the efficiency of the machine under test. From $\dot{E}_o$ and $\dot{I}_6$ the power input may be found, for

$$W = ei + e^1\, i^1$$

and in this case, where $\dot{E}_o = e\ (N + jO)$ and $\dot{I}_6 = e\ (H + jK_1)$

$$W = e^2\ (NH + OK) = \text{Input.}$$

The output is the product of the secondary terminal E. M. F., e,

and the energy component of the load current, eA, and hence is e^2A. Therefore,

$$\text{Eff} = \frac{\text{Output}}{\text{Input}} = \frac{A}{NH + OK}.$$

This value of efficiency involves all of the calculated values except those for E_0 (real) and those for regulation. It may be checked by means of the stray power or loss method, described and used in Experiment 33. In this case the copper losses in the primary and secondary and the eddy current losses in the conductors are included in the impedance watts and the core loss is given as efficiency is directly obtained.

$$\text{Eff.} = \frac{\text{Output}}{\text{Output + Losses}}$$

In the sample taken, e = 1 and is omitted.

Nh	1.0253	0.8355	0.8053
Ok	0.0003	0.0108	-0.0187
NH + OK	1.0250	0.8247	0.8240
Eff. $\frac{A}{NH + OK}$	97.6%	96.9%	97%
W (Copper)	1200	1200	1200
W (iron)	1220	1220	1220
Total Losses	2420	2420	2420
Total loss in %	.0242	.0242	.0242
Input loss + P	1.0242	.8242	.8242
Eff. P/Input	97.6%	97%	97%

DESIGN OF A CORE TYPE TRANSFORMER

5 K. W. , 60 cycles, $E_p = 110$, $E_s = 220/110$, $I_p = 4.55$ amperes, $I_s = 22.75/45.5$.

Volts per turn for a 5 K. W. transformer from curve page 127 = 1.

Turns on primary = $n_p = \frac{1100}{1} = 1100$.

$$E = 4.44\ f\ n\ 0\ 10^{-8}$$

$$\phi = \frac{1100 \times 10^{8}}{4.44 \times 60 \times 1100} = 376{,}000 \text{ lines.}$$

Assuming B = 60,000 lines per square inch,

Cross section area of core = $\frac{376{,}000}{60{,}000} = 6.28$ sq. in. (net)

$1.1 \times 6.28 = 6.9$ sq.in. (gross)

$6.9 = 2.63$ inches.

Make outside diameter of core 3" x 3" in cross section.

$9 - 6.9 = 2.1$ sq. in. (area of 4 corners cut out)

$\frac{2.1}{4} = .52$ sq. in. (area of one corner)

$\sqrt{.52} = .72"$

Assume current density = 1000 amperes per square inch.

$\frac{4.55}{1000} = .00455$ sq. in. = #12 B & S wire for primary.

$\frac{22.75}{1000} = .02275$ sq. in. = #5 B & S wire for secondary.

Make ratio of height of window to width of window equal approximately to 4.

Primary consists of 2 coils, 550 turns each, 5 turns deep, 110 turns high.

Secondary consists of 2 coils, 110 turns each, 2 turns deep, 55 turns high.

(Pri.) Diameter #12 B. & S., D. C. C. = .093". Area = .0068 sq.in.

(Sec.) Diameter #5 B. & S., D. C. C. = .194". Area = .0295 sq.in.

Distance between coils = 0.2 inches.

Thickness of insulating tape = 0.02 inches.

Mean length per turn of secondary = 2π (1.5 + .325 + .234) = 12.9".

Mean length per turn of primary = 2π (1.5 + .325 + .2 + .278) = 17.4".

Res. per 1000 ft. #12 B. & S. at 60 C = 1.846 ohms.

Res. per 1000 ft. #5 B. & S. at 60 C = .3642 ohms.

Total Pri. Res. = $1100 \frac{17.4}{12} 1.846 \times 10^{-3} = 2.94$ ohms.

Total Sec. Res. = $220 \frac{12.9}{12} .3642 \times 10^{-3} = .086$ ohms.

Net volume of core = 6.28 (11.2 x 2 + 9.3 x 2) = 258 cu. in.

Weight of core = $\frac{258}{1728}$ 490 = 73 pounds.

(From curve) Watts core loss per pound of core at density (B =60,000) and 60~ = .82 watts.

$$73 \times .82 = 59.8 \text{ watts (core loss)}$$

$$\frac{59.8}{5000} = 1.2\%.$$

$$I_{C.L.} = \frac{59.8}{1100} = .0544 \text{ amp.} = 1.2\% \text{ of full load.}$$

Mean length of magnetic path in iron = 2(11.2 + 9.3) = 41".

Assume length of path in air per joint = .0015".

Assume length of path in air for 4 joints = .006".

(Air) A. T. = .313 B = .313 x 60,000 x .006 = 113.

(From curve) A. T. per inch length of magnetic path in iron = 7.

$$7 \times 41 = 287 \text{ A.T.}$$

$$\text{Total A. T.} = 113 + 287 = 400 \text{ A. T.}$$

Maximum magnetizing current = $\frac{400}{1100}$ = .363 amp.

$$I_{Mag.} = \frac{.363}{\sqrt{2}} = .257 \text{ amp.} = 5.65\% \text{ of full load.}$$

$$I_o = \sqrt{I^2_{Mag.} + I^2_{C\ L}} = \sqrt{.257^2 + .0544^2} = .262 \text{ amp.} = 5.76\ \%.$$

$$L = \frac{4\ \pi\ N'^2}{L'} \left(\frac{\overline{X}\lambda}{3} + \frac{Y\lambda}{3} + g\lambda + g\rho\right) 10^{-9}$$

N' = Total Pri. turns = 1100.

L' = Height of coil = 10.7" = 27.2 cm.

$\overline{X}$ = Width of Pri. = .545" = 1.384 cm.

Y = Width of Sec. = .468" = 1.19 cm.

λ = Twice depth of core = 6." = 15.24 cm.

g = Distance between Pri, and Sec. = .2" = .508 cm.

ρ = Twice mean length of coils around one leg = 11.25" x 2 = 22.5" = 57.1 cm.

$$L = \frac{4\ \pi \times 1100^2}{27.2} \left(\frac{1.384 \times 15.24}{3} + \frac{1.19 \times 15.24}{3} + .508 \times 15.24 + .508 \times 57.1\right) 10^{-9} = 27{,}800{,}000 \times 10^{-9} \text{ Henrys.}$$

$$X_t = 2\ \pi\ f\ L = 10{,}500{,}000{,}000 \times 10^{-9} = 10.5 \text{ ohms.}$$

$$\%\ X_t = \frac{4.55 \times 10.5}{1100} = 4.34\ \%.$$

$$\%\ X_p = \%\ X_s = 2.17\ \%.$$

Res. of Pri. at 15 C = $1100\ \frac{17.4"}{12}\ 1.559\ \ 10^{-3}$ = 2.484 ohms.

Res. of Sec. at 15 C = $220\ \frac{12.9}{12}\ .3076\ \ 10^{-3}$ = .0726 ohms.

Lbs. per ohm of Pri. at 15 C = 12.678.

Lbs. per ohm of Sec. at 15 C = 325.01.

Weight of copper in Pri. = 2.484 x 12.678 = 31.5 lbs.

Weight of copper in Sec. = .0726 x 325.01 = 23.6 lbs.

55.1 lbs. Total.

If cost of Cu = 22 ¢. per lb.

Total cost = 55.1 x $.22 = $12.1

Weight of iron core = 73 lbs.

If cost of iron = 6 ¢. per lb.

Total cost = 73 x \$.06 = \$4.35

Volume of total Pri. winding = 1100 x 17.4 x .00513 = 98.3 cu.in.

Volume of total Sec. winding = 220 x 12.9 x .026 = 73.7 cu. in.

$$\text{Space factor} = \frac{1100 \times .00513 + 220 \times .026}{3.3 \times 11.2} = .308.$$

Total radiating surface of Pri. = 4 (17.4 x 10.7 + 17.4 x .545) =
782 sq. in.

Total radiating surface of Sec. = 4 (12.9 x 10.7 + 12.9 x .468) =
576 sq. in.

Radiating surface of core = 41 x 4 x 3 = 487 sq. in.

Full load I^2R loss in Pri. = $\overline{4.55}^2 \times 2.94 = 60.8$ watts.

Full load I^2R loss in Sec. = $\overline{22.75}^2 \times .086 = 44.5$ watts.

105.3 watts total.

Watts radiating from Pri. per sq. in. = $\frac{60.8}{782} = .078$ watts.

Watts radiating from Sec. per sq. in. = $\frac{44.5}{576} = .077$ watts.

$\frac{59.8}{487} = .123$ watts per sq. in. radiating from core.

$$\text{Eff} = \frac{\text{Output}}{\text{Output} + I^2R \text{ losses} + \text{core losses.}}$$

$$1/4 \text{ load Eff.} = \frac{1250}{1250 + \frac{105.3}{16} + 59.3} = 95\ \%$$

$$1/2 \text{ load Eff.} = \frac{2500}{2500 + \frac{105.3}{16} + 59.3} = 96.6\%$$

$$3/4 \text{ load Eff.} = \frac{3750}{3750 + 9/16\ 105.3 + 59.3} = 96.9\%$$

$$\text{Full load Eff.} = \frac{5000}{5000 + 105.3 + 59.8} = 96.9\%$$

$$1\ 1/4 \text{ load Eff.} = \frac{6250}{6250 + \frac{25}{16}\ 105.3 + 59.8} = 96.5\ \%.$$

$$\text{All day Eff.} = \frac{\text{Output}}{\text{Output + losses}}.$$

Output for 3 hours = 3 x 5000 = 15,000 watts.

I^2R loss for 3 hours = 3 x 105.3 = 3.6 watts.

Core loss for 24 hours = 24 x 59.8 = 1438 watts.

$$\text{Eff.} = \frac{15000}{15000 + 316 + 1438} = 89.6\ \%.$$

Approximate volume of coil = 74.2 x 18.2 = 1345 cu. in. = 5.84 gals.

$$\%\ r_p = \frac{4.55 \times 2.94}{1100} = .01216 \qquad \%\ a = \%\ I_{CL}$$

$$\%\ r_s = \frac{22.75 \times .086}{220} = .0089 \qquad \%\ b = -\%\ I_M$$

Calculations by Complex Quantities (Simplified)

	$P.F._3 = .8$ Lead	$P.F._3 = 1$	$P.F._3 = .8$ Lag
e	1.	1.	1.
i	.8	1.	.8
i'	.6	0	-.6
$i\ r_s$	.00712	.0089	.00712
$i'x_s$	.013	0	-.013
$i'r_s$	.00534	0	-.00534
$i\ x_s$	.01735	.0217	.01735
e_i	.99412	1.0089	1.02012
$e_i a$	.01192	.0121	.01224
$e'_i a$	.00027	.00026	.000144
$e'_i b$	-.00128	-.001225	-.00068
$e_i b$	-.0562	-.057	-.0577
i_{oo}	.0132	.013325	.01292

	$P.F._3 = .8$ Lead	$P.F._3 = 1$	$P.F._3 = .8$ Lag
i'_{oo}	-.05593	-.05674	-.057556
e'_i	.02269	.0217	.01201
i_p	.8132	1.013325	.81292
i'_p	.54407	-.05674	-.657556
$i_p r_p$	.00988	.01233	.009876
$i'_p x_p$	.0118	-.01232	-.01427
$i'_p r_p$	.00662	-.00069	-.008
$i_p x_p$	.01764	.022	.01763
e_o	.9922	1.03355	1.044265
e'_o	.04671	.04301	.02164
E_o	.994	1.034	1.044
Reg.	-.006	.034	.044
$e_o i_p$	.807	1.048	.849
$e'_o\ i'_p$	.0254	-.00244	-.0142
P	.8324	1.04556	.8348
e i	.8	1.	.8
Eff. $= \frac{e\ i}{p}$	.961	.958	.959
I_p	.976	1.014	1.044
$P.F._p = \frac{P}{E_o I_p}$	.859	.996	.765

CASE

Cross section inside = 6.3" x 6.5" + $\pi \times \overline{3.25}^2$ = 74.2 sq. in.

Height of transformer + blocks = 11.2 + 6 + 1 = 18.2".

Height of case = 5/4 x 18.2 = 22.75".

Thickness of case = 1/4 in.

Area of radiating surface (vertical only) = $(2 \times 6.3 + 2\pi \times 3.5)$

$22.75 = 786$ sq. in.

Total watts radiating from case = I^2R loss + core loss,

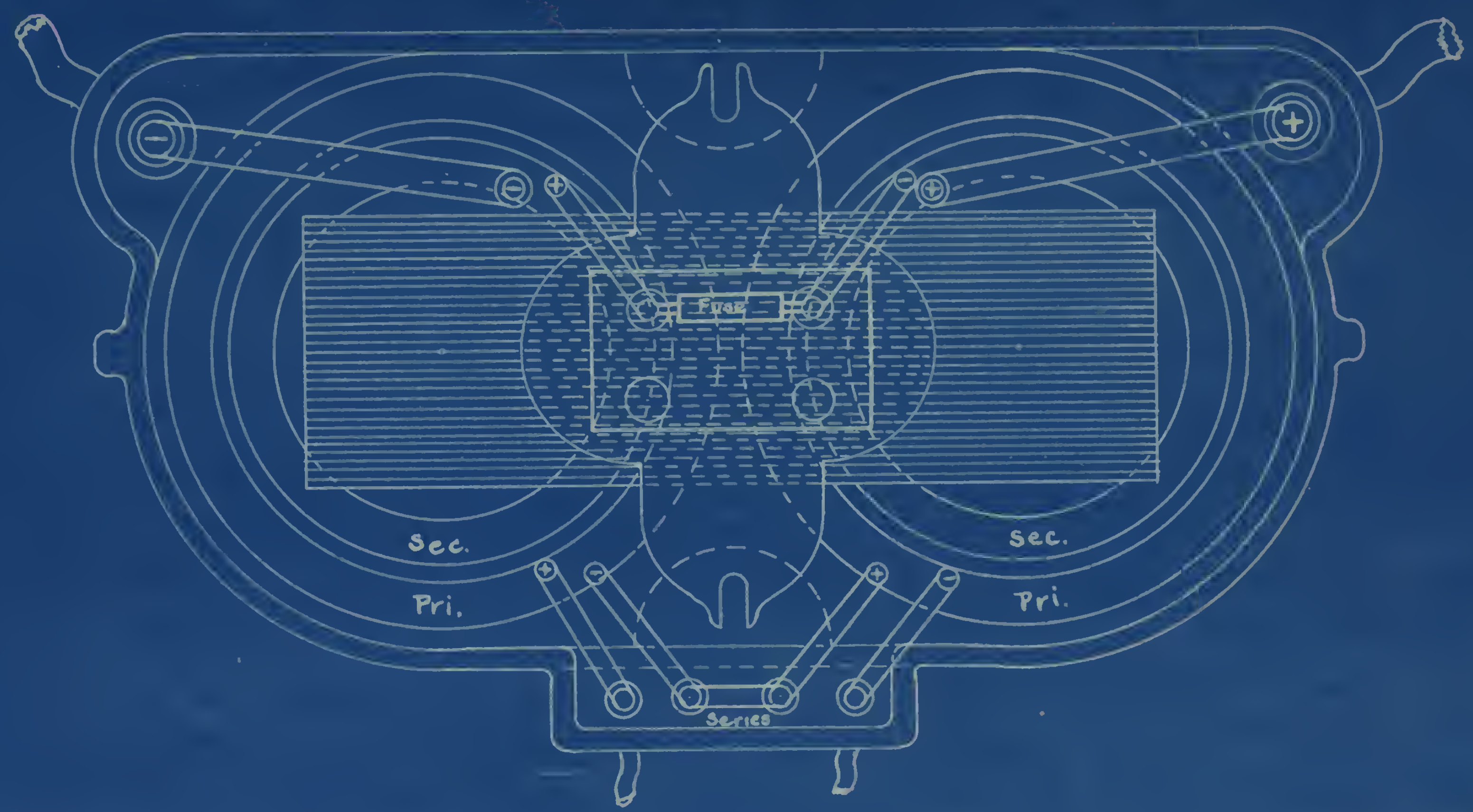
$= 105.3 + 59.8 = 165.1$ (full load)

Watts radiated per sq. in. of surface = $\frac{165.}{786} = .21$ watts per sq.in.

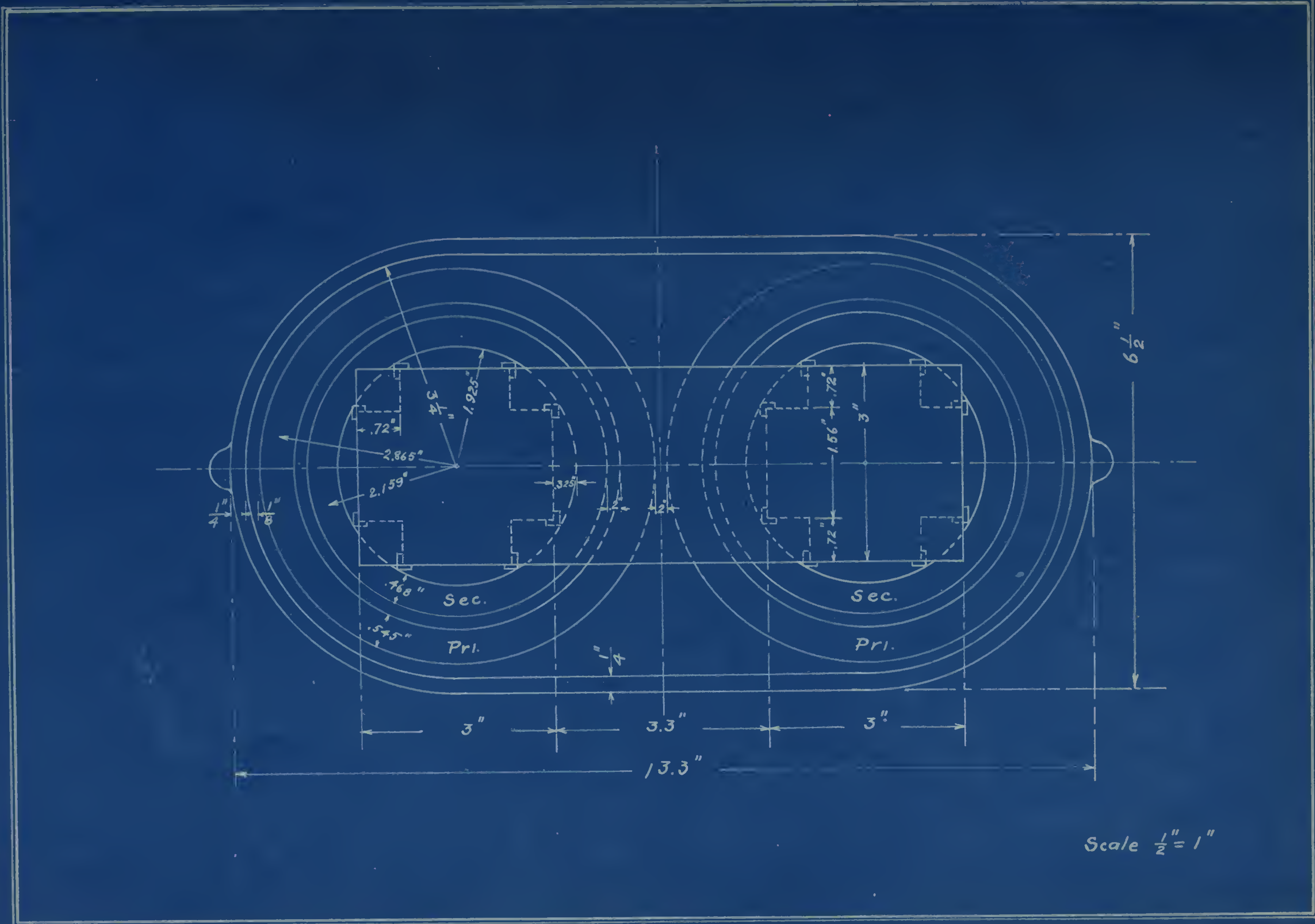
Approximate volume of case = $786 \times 1/4 + 2 \times 75 \times 1/4 = 236$ cu.in.

Approximate weight of case = $\frac{236}{1728}\ 490 = 67$ lbs.

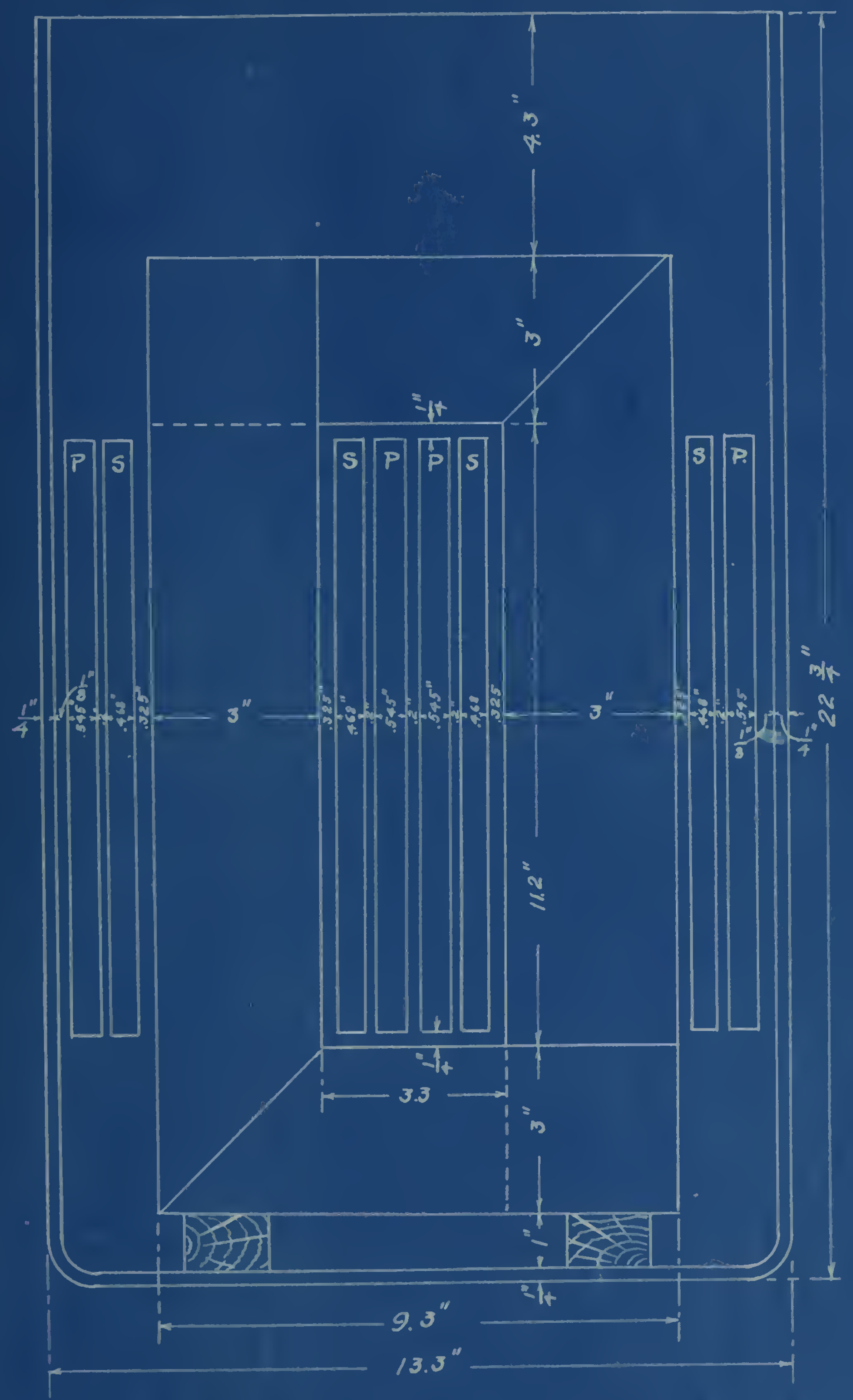
Approximate cost of case at \$.03 per lb. of iron = \$2.01.



Scale: $\frac{1}{2}'' = 1''$



Sec.
Pri.
Sec.
Pri.
3"
3.3"
3"
13.3"
$6\frac{1}{2}$"
$3\frac{1}{4}$"
1.925"
2.865"
2.159"
.72"
.72"
.72"
1.56"
3"
.325"
.2"
.2"
.468"
.545"
$\frac{1}{4}$"
$\frac{1}{8}$"
$\frac{1}{4}$"
Scale $\frac{1}{2}$" = 1"



Scale 1" = 3"

V THE INDUCTION MOTOR

In writing on the design of the induction motor, no theory will be given in this thesis but reference will here be made to Electrical Engineering by C. P. Steinmetz, pages 352-372, on which pages will be found the theory and method of applying the constants that will be calculated in this thesis.

MAGNETO MOTIVE FORCE

It has been shown that the armature of a multiphase alternator when supplying current to a load produces a magneto-motive force, constant in value and fixed in space, that is fixed with reference to the stationary part, or field, of the machine, but revolving with reference to the armature at synchronous speed. Should the armature be held in a fixed position and be supplied with a polyphase current a magneto motive force will be set up constant in intensity but revolving in space at a speed corresponding to synchronism with the sourse of supply.

This then gives the condition for the production of a magneto motive force in the polyphase motor. The field or stator of a polyphase motor is wound in the same manner as the armature of a polyphase alternating current generator and to these windings a polyphase current is supplied the result being a magneto motive force is set up therein, the value of which is expressed by the same equation as represents the armature reactions for a polyphase alternator. That is the magneto motive force per pole for a polyphase induction motor is expressed as follows,

$$M. M. F. = \frac{n}{2}\sqrt{2}\, I_m N$$

where n = the number of phases,

I_m = the effective current per phase,

N = Turns per pole per phase.

MAGNETIZING CURRENT

In calculating the magnetizing current the total ampere turns per pole are found as for any of the previous machines and this value is equated to $\frac{n}{2}\sqrt{2}\, I_m$ N----. In this case I_m = the effective value of the magnetizing current provided the ampere turns per pole are calculated in effective values.

LENGTH OF MAGNETIC PATH

The length of the magnetic path per pole for a four pole induction motor is shown in figure 71. As shown in the figure the length of magnetic circuit per pole includes,

(a) Once across the air gap.

(b) Once through the stator teeth.

(c) Once through the rotor teeth.

(d) From top of stator teeth to line bisecting the pole.

(e) From bottom of rotor teeth to line bisecting the pole.

The ampere turns for the iron part of the circuit are obtained from an iron curve while the ampere turns for the air gap are found from the equation,

A. T. (for gap) = $.313 \beta$ L, where,

β = density in lines per square inch.

L = length of gap in inches.

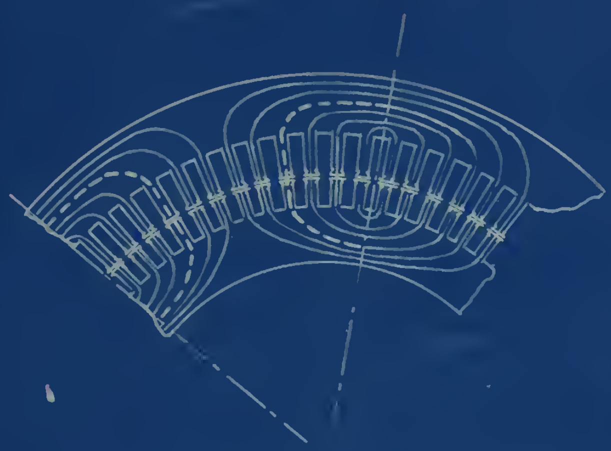


Fig. 71

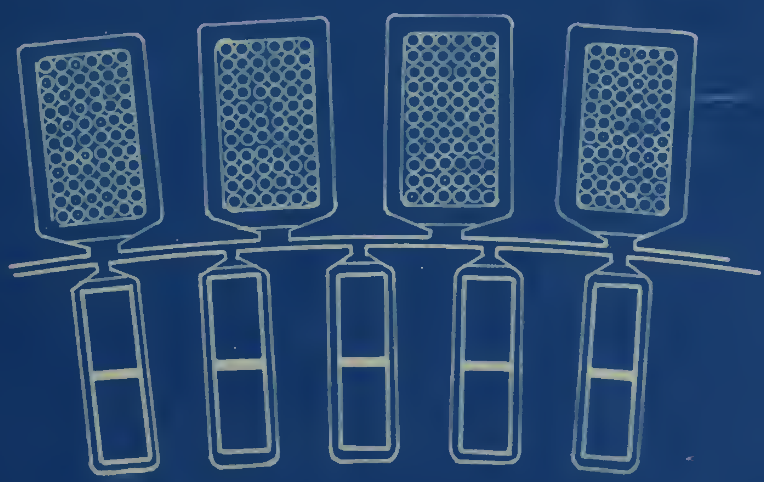


Fig. 72

It must be remembered that in each case β is the maximum value of flux density and hence the value of ampere turns corresponding will be maximum and may be reduced to effective value by dividing by the $\sqrt{2}$.

DIAMETER

As a trial value for the outside diameter of the rotor the following have been found to give good results.

(a) For 60 cycle machines.

The diameter per pole should be about 3".

(b) For 25 cycle machines.

The diameter per pole should be about 7 1/2".

SLOTS PER POLE

The number of slots should be such as will make an equal number of slots per pole per phase and may be found by assuming the slot pitch of the machine. Slot pitch for small machines may be taken as about 1/2 inch and for large machines one inch. A typical stator and rotor slot is shown in figure 72.

SLOT INSULATION

Slot insulation varies with the voltage and kind of material used. The following table gives approximate values of slot lining insulation.

TABLE

Terminal volts	Thickness of slot lining
500	.9 m m.
1000	1.4 m m.
2000	2.3 m m.
4000	3.3 m m.
8000	4.7 m m.

The flux per pole for the induction motor can be found by application of the formula,

$$E = 4.44 \times f \times \varphi \times 10^{-8}$$

where E = impressed voltage per phase,

f = the frequency of this voltage,

N = the turns per phase.

In order to determine the proper number of conductors on turns to be used per phase the following trial value will be given. Wind on the stator 100 ampere conductors per inch of periphery per $\frac{1}{100}$ inch length of air gap. For small motors make the length of air gap equal .02 inch and for the largest motors the gap length should never be greater than .10 inch.

Assuming a gap of .03 inch there will then be 300 ampere conductors on the periphery of the machine at full load and dividing 300 by full load current per winding the conductors per inch of periphery are obtained.

FLUX DENSITY

The following tables give approximate densities to use.

60 Cycles Small Machines.

Section	Density lines per square inch
Air gap	20,000
Stator teeth	40,000
Rotor teeth	40,000
Stator body	60.000
Rotor body	60.000

60 Cycles Large Machines.

Air gap	30.000
Stator teeth	50.000 - 60.000
Rotor teeth	50.000 - 60.000
Stator body	60.000
Rotor body	60.000

25 Cycles Large Machines.

Air gap	40.000
Stator teeth	90.000
Rotor teeth	90.000
Stator body	90.000 - 100.000
Rotor body	90.000 - 100.000

AREA OF THE AIR GAP

Since the length of the air gap is very short in almost all induction motors, the area of the gap per pole may be assumed to be equal to the area of the stator teeth per pole.

This is due to the fact that the rotor slots are very

nearly closed and since the gap is very short there will be little if any fringing of the flux. From the above table it is evident the length of the machine can be fixed so as to obtain the proper density in the air gap.

CORE LOSS

The core loss of the induction motor may be found by the following process.

(a) Calculate flux density in and weight of stator teeth.

(b) Calculate flux density in and weight of stator body.

From curve 32 find the watts lost per pound and then the total stator loss in stator.

(c) Add to this loss 20% for rotor loss and the sum will give the total hysteresis and eddy current losses of the motor.

CORE LOSS CURRENT

The core loss current per phase will then be total core loss per phase divided by the voltage per phase and in percent will be the core loss current per phase divided by full load current per phase or, $\frac{I_c}{I_s}$ = % core loss current.

The exciting or no load current will then be,

$$I_{ex.} = \sqrt{I_c^2 + I_m^2}$$, where I_m = the magnetizing current per phase as previously calculated.

STATOR RESISTANCE

The current density in the stator may be assumed as 2000 ampere per square inch, which gives the size of the stator wire as

follows,

$$\text{Area of wire in square inches} = \frac{\text{Full load current}}{2000}.$$

The length of one turn in inches on the stator may be taken as,

$$2\,L + 10\,\frac{D}{P}\quad,\ \text{where}$$

L = length of stator in inches,

D = diameter of rotor in inches,

P = number of poles.

Hence from the above the resistance per phase winding may be calculated. The resistance in percent then equals,

$$\frac{\text{Resistance per phase x Full load current}}{\text{Voltage per phase}}.$$

ROTOR

The current flowing in each rotor bar of a squirrel cage induction motor at full load may be found by the following equation,

$$\frac{I_s\,Z_s}{Z_r} = I_r\ ,$$

where I_s = current in stator windings at full load,

Z_s = total number of conductors on stator,

Z_r = total bars on the rotor,

I_r = full load current in each rotor bar.

The size of the bar may next be found by assuming a current density of 2000 to 2500 amperes per square inch or the area of rotor bar in square inches will equal $\frac{I_r}{2500}$.

Now the rotor bars are short circuited by two rings one at each end of the rotor and therefore the resistance of the whole rotor must be calculated.

Let fugure 73 represent a squirrel cage rotor.

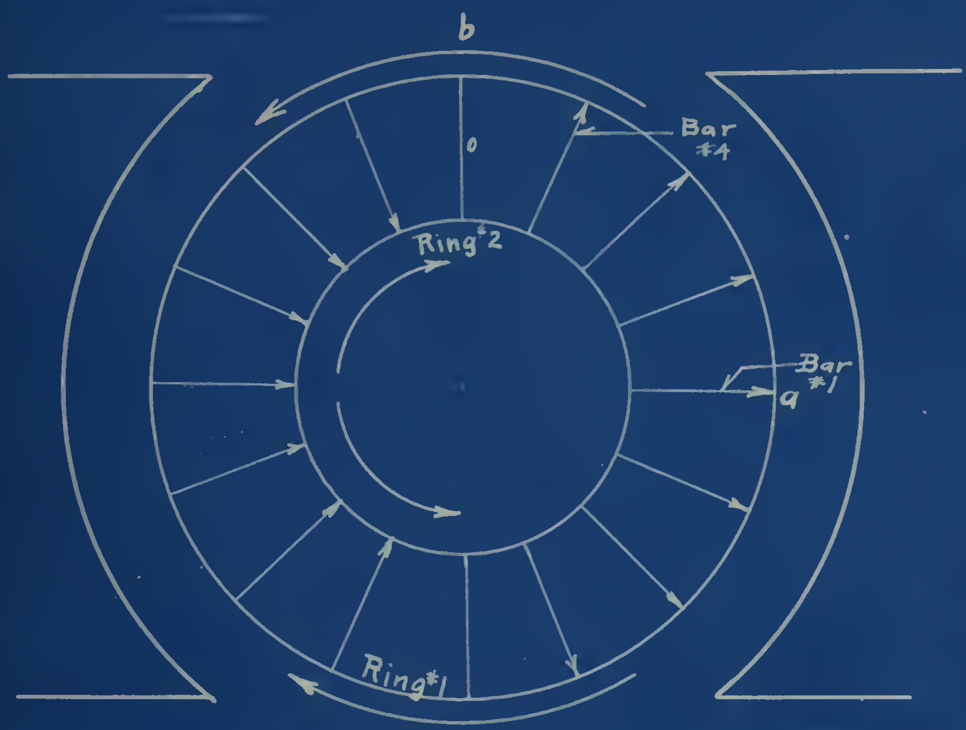


Fig. 73

Let the revolving field be represented by the two definite poles.

Let the outside circle represent one ring and the inside circle the other and let the radial lines represent the rotor conductors. Then at any instant the arrows on the conductors and at the side of the ring represent the flow of current.

Let Z_r = total conductors on the rotor.

Z_r = either 1/2 the stator slots $\pm$ 1, or

= the stator slots $\pm$ 1,

and usually there is not more than two bars per slot.

Let I_r = the effective current per bar = $\frac{Z_s\ I_s}{Z_r}$ and is assumed to be the same value in all bars on the rotor. From the figure it is evident that the current in the rotor bars on one side of the rotor flows from ring 2 to ring 1 and on the opposite side from ring #1 toward ring #2. It is also evident that the current in bar #1 is not in phase with the current in bar #3 or #4, and therefore the current in the ring section (a - b) will be the vector sum of the current from bar 1, 2, 3, 4 etc. or all bars between point a and point b.

For a two pole machine it is evident that 1/4 the conductors furnish current to any section of the ring and if we add these currents properly as regards phase relation we have, that the current in any section of the ring of a two pole machine will be the vector sum of all bar currents feeding any section of the ring.

$$I_R = \frac{Z_r\ I_r}{4}\ \frac{2}{\pi} = \frac{Z_r\ I_r}{2\pi}$$ for a two pole machine or,

$I_R = \frac{Z_r I_r}{2\pi P}$ for P pairs of poles, where I_R is the current in any section of the ring.

Now let D = the diameter of the ring in inches and let ω_r = the resistance of the ring per inch length.

Then the loss in one ring will be,

$$I_R^2 \pi D \omega_r$$

and the loss in both rings will be,

$$2 I_R^2 \pi D \omega_r.$$

Substituting for I_R its value $\frac{Z_r I_r}{2\pi P}$, the loss in both rings becomes,

$$\frac{Z_r^2 I_r^2 D \omega_r}{2 P^2 \pi}.$$

Now let ω_b = resistance of one bar. Then the total loss in the bars will be,

$Z_r I_r^2 \omega_b$, and the total loss in the rotor is,

$$\frac{Z^2 Ir^2 D \omega_r}{2 P^2 \pi} + 2 I_r^2 \omega_b$$

, or representing this loss by L,

$$L = I_r^2 Z_r \left[\frac{Z_r D \omega_r}{2 P^2 \pi} + \omega_b \right]$$

The effective resistance of the rotor reduced to equivalent stator resistance per phase is for an n phase motor r_2.

$r_2 = \frac{L}{n I^2}$, where I = full load current of the stator per phase.

The rotor rings may be made to carry a current density of from 2000 to 3000 amperes per square inch and are usually made of brass which has a resistance about three times that of copper. The resistance is about $\frac{.666}{10^6}$ per inch cube. For brass about $\frac{1.998}{10^6}$ per inch cube.

STATOR AND ROTOR REACTANCE

The stator and rotor reactances are found by the proper application of formula 102 . That is the formula is applied at least twice to the stator and the average result taken as stator reactance.

The stator slot is first placed so that the opening at the air gap is exactly over the center of the rotor tooth and secondly so that the stator slot opening is exactly above the slot opening of the rotor, then the average reactance as calculated in these two positions is taken as the primary reactance.

To get reactance in % multiply the calculated value by the full load stator current and divide by the voltage per phase. The same process is used for the rotor except that when the rotor reactance X_s is calculated in ohms, it is reduced to equivalent stator reactance by multiplying X_s by the ratio of transformation squared.

Or X_s equivalent stator $= X_s \dfrac{I_r^2}{I_s^2}$

And X_s'equivalent in % $= \dfrac{X_s \frac{I_r^2}{I_s^2} I_s}{E_s}$ where E_s = the stator voltage per phase.

SUMMATION OF CONSTANTS.

Let r_1 = primary resistance per phase in ohms as calculated.

Then % primary resistance $= \dfrac{I_s \, r_1}{E_s}$.

Let r_2 = rotor resistance reduced to equivalent stator

resistance as given above.

Then $\frac{r_2 \; I_s}{E_s}$ = rotor resistance in % reduced to primary.

Let I_c = core loss current, then

(a) the conductance in % is,

$\frac{I_c}{I_s}$ where I is the full load stator current.

Let I_m = magnetizing current, then

(b) the susceptance in % is $\frac{I_m}{I_s}$.

Let x_1 = stator reactance in ohms, then the stator reactance in % will be $\frac{X_1 \; I_s}{E_s}$.

Let X_2'= rotor reactance in ohms reduced to equivalent stator reactance, then the equivalent stator reactance will be,

$$\frac{X_2' \; I_s}{E_s}.$$

The above constants are all that are necessary to calculate the preformance curves of an induction motor by the analytical method as mentioned on page .

A study of the derived curves and the factors governing them will lead to the satisfactory design of an induction.